

Lecture 07b Initialization

DL4DS – Spring 2025

Agenda

- Project: <u>Project ideas</u>, Teams, ...
- Quick tips on how to read a research paper
- Model Initialization
- Example code

Where we are



=== Foundational Concepts ===

- √ 02 -- Supervised learning refresher
- √ 03 -- Shallow networks and their representation capacity
- √ 04 -- Deep networks and depth efficiency
- ✓ 05 -- Loss function in terms of maximizing likelihoods
- ✓ 06 Fitting models with different optimizers
- ✓ 07a Gradients on deep models and backpropagation
- 07b Initialization to avoid vanishing and exploding weights & gradients
- 08 Measuring performance, test sets, overfitting and double descent
- 09 Regularization to improve fitting on test sets and unseen data

=== Network Architectures and Applications ===

- 10 Convolutional Networks
- 11 Residual Networks
- 12 Transformers
- Large Language and other Foundational Models
- Generative Models
- Graph Neural Networks
- ...

How to Read Research Papers

Reading Research Papers, Accelerating your Efficiency

- Compile list of papers (research papers, Medium posts, Blogs, etc.). You can use tools like Zotero or Mendeley to organize your bibliography.
- Skip around the list.
 - a. Glance through the list, maybe read only 10% of each paper.
 - b. Weed out duds.
 - c. Pick one of the papers to dive into.
- 3. 15-20 papers gives you basic understanding maybe enough to implement. 50-100 papers give you very solid understanding, can start research.
- 4. Shoot for reading 1-2 papers deeply per week. ⇒ Reading Group!

How to Read Research Papers - II

How to Read a Paper

Do it in multiple passes.

- 1. Title, abstract and figures. Especially the key figure(s).
- 2. Intro, Conclusions, Figures then skim the rest
- 3. Maybe skim or skip related work. Sometimes authors try to cite people that may be reviewers.
- 4. Read the paper but skim/skip the math.
- 5. Read whole thing but skip parts that don't make sense. Often even authors don't know what will make the biggest impact until later.
- 6. Go Deeper: Re-derive the math from memory.
- 7. Go Deeper: Re-implement the network from scratch.

How to Read Research Papers - III

Some questions to ask yourself.

- What did authors try to accomplish?
- What were the key elements of the approach?
- What can you use yourself?
- What other references do you want to follow?

Optional Exercise

Look up "<u>Deep Residual Learning for Image Recognition</u>" on scholar.google.com.

Take 7 minutes to scan and read the paper in multiple passes as previously discussed. Highlight key points and graphs.

Take a few minutes and jot down the key takeaways.

Model Initialization

- The need for weights initialization
- Expectations Refresher
- The He (Kaiming) Initialization

Initialization

Consider standard building block of NN in terms of pre-activations:

$$egin{aligned} \mathbf{f}_k &= oldsymbol{eta}_k + oldsymbol{\Omega}_k \mathbf{h}_k \ &= oldsymbol{eta}_k + oldsymbol{\Omega}_k \mathbf{a}[\mathbf{f}_{k-1}] \end{aligned}$$

- How do we initialize the biases and weights?
- Equivalent to choosing starting point in our gradient descent searches

Forward Pass

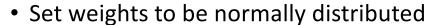
Consider standard building block of NN in terms of pre-activations:

$$\mathbf{f}_k = \boldsymbol{\beta}_k + \boldsymbol{\Omega}_k \mathbf{h}_k$$

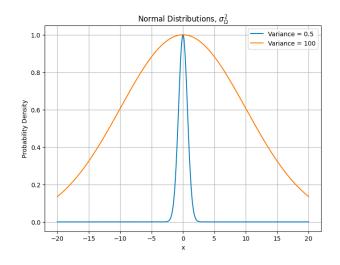
= $\boldsymbol{\beta}_k + \boldsymbol{\Omega}_k \mathbf{a}[\mathbf{f}_{k-1}]$

Set all the biases to 0

$$\boldsymbol{eta}_k = \mathbf{0}$$



- mean 0
- variance σ_0^2

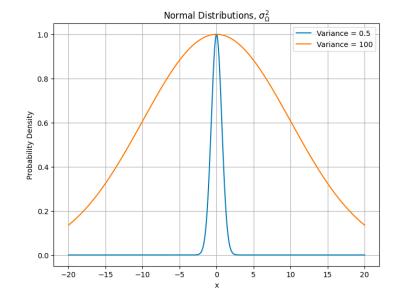


- What will happen as we move through the network if σ_{Ω}^2 is very small?
- What will happen as we move through the network if σ_{Ω}^2 is very large?

Backward Pass

$$\frac{\partial \ell_i}{\partial \mathbf{f}_{k-1}} = \mathbb{I}[\mathbf{f}_{k-1} > 0] \odot \left(\mathbf{\Omega}_k^T \frac{\partial \ell_i}{\partial \mathbf{f}_k} \right), \qquad k \in \{K, K-1, \dots 1\}$$
 (7.13)

- What will happen as we propagate backwards through the network if σ_{Ω}^2 is very small?
- What will happen as we propagate backwards through the network if σ_{Ω}^2 is very large?



Initialize weights to different variances

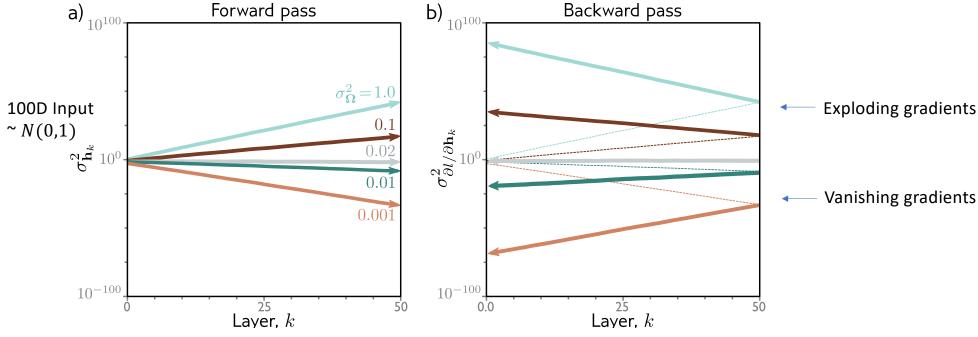


Figure 7.4 Weight initialization. Consider a deep network with 50 hidden layers and $D_h = 100$ hidden units per layer. The network has a 100 dimensional input \mathbf{x} initialized with values from a standard normal distribution, a single output fixed at y = 0, and a least squares loss function. The bias vectors $\boldsymbol{\beta}_k$ are initialized to zero and the weight matrices Ω_k are initialized with a normal distribution with mean zero and five different variances $\sigma_{\Omega}^2 \in \{0.001, 0.01, 0.02, 0.1, 1.0\}$. a)

How do we initialize weights to keep variance stable across layers?

$$\mathbf{f}' = oldsymbol{eta} + \mathbf{\Omega} \mathbf{h} \ \mathbf{h} = \mathbf{a}[\mathbf{f}],$$

Definition of variance:

$$\sigma_{f'}^2 = \mathbb{E}[(f_i' - \mathbb{E}[f_i'])^2]$$

Agenda

- The need for weights initialization
- Expectations Refresher
- The He (Kaiming) Initialization

Expectations

$$\mathbb{E}\left[g[x]\right] = \int g[x]Pr(x)dx,$$

Interpretation: what is the average value of g[x] when taking into account the probability of x?

Consider discrete case and assume uniform probability so calculating g[x] reduces to taking average:

$$\mathbb{E}\left[g[x]\right] \approx \frac{1}{N} \sum_{n=1}^{N} g[x_n^*]$$
 where $x_n^* \sim Pr(x)$

Common Expectation Functions

Function $g[\bullet]$	Expectation
x	mean, μ
x^k	kth moment about zero
$(x-\mu)^k$	kth moment about the mean
$(x-\mu)^2$	variance
$(x-\mu)^3$	skew
$(x-\mu)^4$	kurtosis

Table B.1 Special cases of expectation. For some functions g[x], the expectation $\mathbb{E}[g[x]]$ is given a special name. Here we use the notation μ_x to represent the mean with respect to random variable x.

Rules for manipulating expectation

$$\begin{split} \mathbb{E}\Big[k\Big] &= k \\ \mathbb{E}\Big[k \cdot \mathbf{g}[x]\Big] &= k \cdot \mathbb{E}\Big[\mathbf{g}[x]\Big] \\ \mathbb{E}\Big[\mathbf{f}[x] + \mathbf{g}[x]\Big] &= \mathbb{E}\Big[\mathbf{f}[x]\Big] + \mathbb{E}\Big[\mathbf{g}[x]\Big] \\ \mathbb{E}\Big[\mathbf{f}[x]g[y]\Big] &= \mathbb{E}\Big[\mathbf{f}[x]\Big] \mathbb{E}\Big[\mathbf{g}[y]\Big] \quad \text{if} \quad x, y \quad \text{independent} \end{split}$$

Agenda

- The need for weights initialization
- Expectations Refresher
- The He (Kaiming) Initialization

$$\mathbf{h} = \mathbf{a}[\mathbf{f}], \ \mathbf{f}' = oldsymbol{eta} + oldsymbol{\Omega} \mathbf{h}$$

Definition of variance:

$$\sigma_{f_i'}^2 = \mathbb{E}[(f_i' - \mathbb{E}[f_i'])^2]$$

Now let's prove:

$$\mathbb{E}\left[(x-\mu)^2\right] = \mathbb{E}[x^2] - \mathbb{E}[x]^2$$

Keeping in mind:

$$\mathbb{E}[x] = \mu$$

Rule 1:
$$\mathbb{E}\Big[k\Big] = k$$
 Rule 2:
$$\mathbb{E}\Big[k \cdot \mathrm{g}[x]\Big] = k \cdot \mathbb{E}\Big[\mathrm{g}[x]\Big]$$
 Rule 3:
$$\mathbb{E}\Big[\mathrm{f}[x] + \mathrm{g}[x]\Big] = \mathbb{E}\Big[\mathrm{f}[x]\Big] + \mathbb{E}\Big[\mathrm{g}[x]\Big]$$
 Def'n
$$\mathbb{E}[x] = \mu$$

$$\mathbb{E}[(x - \mu^2)] = \mathbb{E}[x^2 - 2x\mu + \mu^2]$$

Rule 1:
$$\mathbb{E}\left[k\right] = k$$
 Rule 2:
$$\mathbb{E}\left[k \cdot \mathbf{g}[x]\right] = k \cdot \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Rule 3:
$$\mathbb{E}\left[\mathbf{f}[x] + \mathbf{g}[x]\right] = \mathbb{E}\left[\mathbf{f}[x]\right] + \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Def'n
$$\mathbb{E}[x] = \mu$$

$$\mathbb{E}[(x - \mu^2)] = \mathbb{E}[x^2 - 2x\mu + \mu^2]$$
$$= \mathbb{E}[x^2] - \mathbb{E}[2x\mu] + \mathbb{E}[\mu^2]$$

Rule 1:
$$\mathbb{E}\left[k\right] = k$$
 Rule 2:
$$\mathbb{E}\left[k \cdot \mathbf{g}[x]\right] = k \cdot \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Rule 3:
$$\mathbb{E}\left[\mathbf{f}[x] + \mathbf{g}[x]\right] = \mathbb{E}\left[\mathbf{f}[x]\right] + \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Def'n
$$\mathbb{E}[x] = \mu$$

$$\mathbb{E}[(x - \mu^2)] = \mathbb{E}[x^2 - 2x\mu + \mu^2]$$

$$= \mathbb{E}[x^2] - \mathbb{E}[2x\mu] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[x^2] - 2\mu\mathbb{E}[x] + \mu^2$$

Rule 1:
$$\mathbb{E}\left[k\right] = k$$
 Rule 2:
$$\mathbb{E}\left[k \cdot \mathbf{g}[x]\right] = k \cdot \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Rule 3:
$$\mathbb{E}\left[\mathbf{f}[x] + \mathbf{g}[x]\right] = \mathbb{E}\left[\mathbf{f}[x]\right] + \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Def'n
$$\mathbb{E}[x] = \mu$$

$$\mathbb{E}[(x - \mu^2)] = \mathbb{E}[x^2 - 2x\mu + \mu^2]$$

$$= \mathbb{E}[x^2] - \mathbb{E}[2x\mu] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[x^2] - 2\mu\mathbb{E}[x] + \mu^2$$

$$= \mathbb{E}[x^2] - 2\mu^2 + \mu^2$$

Rule 1:
$$\mathbb{E}\left[k\right] = k$$
 Rule 2:
$$\mathbb{E}\left[k \cdot \mathbf{g}[x]\right] = k \cdot \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Rule 3:
$$\mathbb{E}\left[\mathbf{f}[x] + \mathbf{g}[x]\right] = \mathbb{E}\left[\mathbf{f}[x]\right] + \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Def'n
$$\mathbb{E}[x] = \mu$$

$$\mathbb{E}[(x - \mu^2)] = \mathbb{E}[x^2 - 2x\mu + \mu^2]$$

$$= \mathbb{E}[x^2] - \mathbb{E}[2x\mu] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[x^2] - 2\mu\mathbb{E}[x] + \mu^2$$

$$= \mathbb{E}[x^2] - 2\mu^2 + \mu^2$$

$$= \mathbb{E}[x^2] - \mu^2$$

Rule 1:
$$\mathbb{E}\left[k\right] = k$$
 Rule 2:
$$\mathbb{E}\left[k \cdot \mathbf{g}[x]\right] = k \cdot \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Rule 3:
$$\mathbb{E}\left[\mathbf{f}[x] + \mathbf{g}[x]\right] = \mathbb{E}\left[\mathbf{f}[x]\right] + \mathbb{E}\left[\mathbf{g}[x]\right]$$
 Def'n
$$\mathbb{E}[x] = \mu$$

$$\mathbb{E}[(x - \mu^2)] = \mathbb{E}[x^2 - 2x\mu + \mu^2]$$

$$= \mathbb{E}[x^2] - \mathbb{E}[2x\mu] + \mathbb{E}[\mu^2]$$

$$= \mathbb{E}[x^2] - 2\mu\mathbb{E}[x] + \mu^2$$

$$= \mathbb{E}[x^2] - 2\mu^2 + \mu^2$$

$$= \mathbb{E}[x^2] - \mu^2$$

$$= \mathbb{E}[x^2] - E[x]^2$$

$$\mathbf{f}' = oldsymbol{eta} + \mathbf{\Omega} \mathbf{h} \ \mathbf{h} = \mathbf{a}[\mathbf{f}],$$

$$\sigma_{f'}^2 = \mathbb{E}[(f_i' - \mathbb{E}[f_i'])^2]$$

$$\sigma_{f'}^2 = \mathbb{E}[f_i'^2] - \mathbb{E}[f_i']^2$$

$$\longrightarrow \mathbb{E}\left[(x-\mu)^2\right] = \mathbb{E}[x^2] - \mathbb{E}[x]^2$$

$$\mathbf{f}' = oldsymbol{eta} + \mathbf{\Omega} \mathbf{h} \ \mathbf{h} = \mathbf{a}[\mathbf{f}],$$

$$\sigma_{f'}^2 = \mathbb{E}[(f_i' - \mathbb{E}[f_i'])^2]$$

$$\sigma_{f'}^2 = \mathbb{E}[f_i'^2] - \mathbb{E}[f_i']^2$$

$$\mathbf{f}' = oldsymbol{eta} + \mathbf{\Omega}\mathbf{h}$$

Consider the mean of the pre-activations:

$$\mathbb{E}[f_i'] = \mathbb{E}\left[\beta_i + \sum_{j=1}^{D_h} \Omega_{ij} h_j\right]$$

Rule 1:
$$\mathbb{E}[k] = k$$

Rule 2:
$$\mathbb{E}\Big[k\cdot \mathbf{g}[x]\Big] = k\cdot \mathbb{E}\Big[\mathbf{g}[x]\Big]$$

Rule 3:
$$\mathbb{E}\Big[f[x] + g[x]\Big] = \mathbb{E}\Big[f[x]\Big] + \mathbb{E}\Big[g[x]\Big]$$

Rule 4:
$$\mathbb{E}\Big[\mathrm{f}[x]g[y]\Big] = \mathbb{E}\Big[\mathrm{f}[x]\Big]\mathbb{E}\Big[\mathrm{g}[y]\Big]$$
 if x,y independent

$$\mathbb{E}[f_i'] = \mathbb{E}\left[\beta_i + \sum_{j=1}^{D_h} \Omega_{ij} h_j\right]$$
$$= \mathbb{E}\left[\beta_i\right] + \sum_{j=1}^{D_h} \mathbb{E}\left[\Omega_{ij} h_j\right]$$

Rule 1:
$$\mathbb{E}[k] = k$$

Rule 2:
$$\mathbb{E}\Big[k\cdot \mathbf{g}[x]\Big] = k\cdot \mathbb{E}\Big[\mathbf{g}[x]\Big]$$

Rule 3:
$$\mathbb{E}\Big[f[x] + g[x]\Big] = \mathbb{E}\Big[f[x]\Big] + \mathbb{E}\Big[g[x]\Big]$$

Rule 4:
$$\mathbb{E}\Big[\mathrm{f}[x]g[y]\Big] = \mathbb{E}\Big[\mathrm{f}[x]\Big]\mathbb{E}\Big[\mathrm{g}[y]\Big]$$
 if x,y independent

$$\mathbb{E}[f_i'] = \mathbb{E}\left[\beta_i + \sum_{j=1}^{D_h} \Omega_{ij} h_j\right]$$

$$= \mathbb{E}\left[\beta_i\right] + \sum_{j=1}^{D_h} \mathbb{E}\left[\Omega_{ij} h_j\right]$$

$$= \mathbb{E}\left[\beta_i\right] + \sum_{j=1}^{D_h} \mathbb{E}\left[\Omega_{ij}\right] \mathbb{E}\left[h_j\right]$$

Rule 1:
$$\mathbb{E}\left[k\right] = k$$

Rule 2:
$$\mathbb{E}\Big[k\cdot \mathbf{g}[x]\Big] = k\cdot \mathbb{E}\Big[\mathbf{g}[x]\Big]$$

Rule 3:
$$\mathbb{E}\Big[f[x] + g[x]\Big] = \mathbb{E}\Big[f[x]\Big] + \mathbb{E}\Big[g[x]\Big]$$

Rule 4:
$$\mathbb{E}\Big[\mathrm{f}[x]g[y]\Big] = \mathbb{E}\Big[\mathrm{f}[x]\Big]\mathbb{E}\Big[\mathrm{g}[y]\Big]$$
 if x,y independent

$$\mathbb{E}[f_i'] = \mathbb{E}\left[\beta_i + \sum_{j=1}^{D_h} \Omega_{ij} h_j\right]$$
$$= \mathbb{E}\left[\beta_i\right] + \sum_{j=1}^{D_h} \mathbb{E}\left[\Omega_{ij} h_j\right]$$

Set all the biases to 0

Weights normally distributed mean 0 variance
$$\sigma_{\Omega}^2$$

$$= \mathbb{E}\left[\beta_i\right] + \sum_{j=1}^{D_h} \mathbb{E}\left[\Omega_{ij}\right] \mathbb{E}\left[h_j\right]$$

$$= 0 + \sum_{j=1}^{D_h} 0 \cdot \mathbb{E}\left[h_j\right] = 0$$

$$\mathbf{f}' = oldsymbol{eta} + \mathbf{\Omega}\mathbf{h}$$
 $\mathbf{h} = \mathbf{a}[\mathbf{f}],$
 $\sigma_{f'}^2 = \mathbb{E}[(f_i' - \mathbb{E}[f_i'])^2]$
 $\sigma_{f'}^2 = \mathbb{E}[f_i'^2] - \mathbb{E}[f_i']^2 = \mathbb{E}[f_i'^2]$

Rule 1:
$$\mathbb{E}[k] = k$$

Rule 2:
$$\mathbb{E}\Big[k\cdot \mathbf{g}[x]\Big] = k\cdot \mathbb{E}\Big[\mathbf{g}[x]\Big]$$

Rule 3:
$$\mathbb{E}\Big[f[x] + g[x]\Big] = \mathbb{E}\Big[f[x]\Big] + \mathbb{E}\Big[g[x]\Big]$$

Rule 4:
$$\mathbb{E}\Big[\mathrm{f}[x]g[y]\Big] = \mathbb{E}\Big[\mathrm{f}[x]\Big]\mathbb{E}\Big[\mathrm{g}[y]\Big]$$
 if x,y independent

$$\sigma_{f'}^2 = \mathbb{E}[f_i'^2] - \mathbb{E}[f_i']^2$$

$$= \mathbb{E}\left[\left(\beta_i + \sum_{j=1}^{D_h} \Omega_{ij} h_j\right)^2\right] - 0$$

Set all the biases to 0

Weights normally distributed mean 0 variance σ_{Ω}^2

Rule 1:
$$\mathbb{E}\left|k\right|=k$$

Rule 2:
$$\mathbb{E}\left[k \cdot \mathbf{g}[x]\right] = k \cdot \mathbb{E}\left[\mathbf{g}[x]\right]$$

Rule 3:
$$\mathbb{E}\Big[f[x] + g[x]\Big] = \mathbb{E}\Big[f[x]\Big] + \mathbb{E}\Big[g[x]\Big]$$

Rule 4:
$$\mathbb{E}\Big[\mathrm{f}[x]g[y]\Big] = \mathbb{E}\Big[\mathrm{f}[x]\Big]\mathbb{E}\Big[\mathrm{g}[y]\Big]$$
 if x,y independent

$$\sigma_{f'}^2 = \mathbb{E}[f_i'^2] - \mathbb{E}[f_i']^2$$

$$= \mathbb{E}\left[\left(\beta_i + \sum_{j=1}^{D_h} \Omega_{ij} h_j\right)^2\right] - 0$$

$$= \mathbb{E}\left[\left(\sum_{j=1}^{D_h} \Omega_{ij} h_j\right)^2\right]$$

Set all the biases to 0

Weights normally distributed mean 0 variance σ_{Ω}^2

Rule 1:
$$\mathbb{E}[k] = k$$

Rule 2:
$$\mathbb{E}\left[k \cdot \mathbf{g}[x]\right] = k \cdot \mathbb{E}\left[\mathbf{g}[x]\right]$$

Rule 3:
$$\mathbb{E}\Big[\mathrm{f}[x] + \mathrm{g}[x]\Big] = \mathbb{E}\Big[\mathrm{f}[x]\Big] + \mathbb{E}\Big[\mathrm{g}[x]\Big]$$

Rule 4:
$$\mathbb{E}\Big[f[x]g[y]\Big] = \mathbb{E}\Big[f[x]\Big]\mathbb{E}\Big[g[y]\Big]$$
 if x,y independent

$$\sigma_{f'}^2 = \mathbb{E}[f_i'^2] - \mathbb{E}[f_i']^2$$

$$= \mathbb{E}\left[\left(\beta_i + \sum_{j=1}^{D_h} \Omega_{ij} h_j\right)^2\right] - 0$$

$$= \mathbb{E}\left[\left(\sum_{j=1}^{D_h} \Omega_{ij} h_j\right)^2\right] \blacktriangleleft$$

$$=\sum_{j=1}^{D_h}\mathbb{E}\left[\Omega_{ij}^2\right]\mathbb{E}\left[h_j^2\right]$$

For all the cross terms, $E\left[\Omega_{ij}\right]=0$ so only the squared terms are left, then use independence.

Set all the biases to 0

Weights normally distributed mean 0 variance σ_{Ω}^2

Rule 1:
$$\mathbb{E}[k] = k$$

Rule 2:
$$\mathbb{E}\left[k \cdot \mathbf{g}[x]\right] = k \cdot \mathbb{E}\left[\mathbf{g}[x]\right]$$

Rule 3:
$$\mathbb{E}\Big[f[x] + g[x]\Big] = \mathbb{E}\Big[f[x]\Big] + \mathbb{E}\Big[g[x]\Big]$$

Rule 4:
$$\mathbb{E}\Big[\mathrm{f}[x]g[y]\Big] = \mathbb{E}\Big[\mathrm{f}[x]\Big]\mathbb{E}\Big[\mathrm{g}[y]\Big]$$
 if x,y independent

$$\sigma_{f'}^2 = \mathbb{E}[f_i'^2] - \mathbb{E}[f_i']^2$$

$$= \mathbb{E}\left[\left(\beta_i + \sum_{j=1}^{D_h} \Omega_{ij} h_j\right)^2\right] - 0$$

$$= \mathbb{E}\left[\left(\sum_{j=1}^{D_h} \Omega_{ij} h_j\right)^2\right]$$

$$= \mathbb{E}\left[\left(\sum_{j=1}^{n} \Omega_{ij} h_{j}\right)\right]$$

$$\begin{split} &= \sum_{j=1}^{D_h} \mathbb{E}\left[\Omega_{ij}^2\right] \mathbb{E}\left[h_j^2\right] \\ &= \sum_{j=1}^{D_h} \sigma_{\Omega}^2 \mathbb{E}\left[h_j^2\right] = \sigma_{\Omega}^2 \sum_{j=1}^{D_h} \mathbb{E}\left[h_j^2\right] \end{split}$$

Because the Ω 's are zero mean, this is the variance.

Set all the biases to 0

Weights normally distributed mean 0 variance σ_0^2

$$\begin{split} \sigma_{f'}^2 &= \sigma_\Omega^2 \sum_{j=1}^{D_h} \mathbb{E}\left[h_j^2\right] \\ &= \sigma_\Omega^2 \sum_{j=1}^{D_h} \mathbb{E}\left[\mathrm{ReLU}[f_j]^2\right] \\ &= \sigma_\Omega^2 \sum_{j=1}^{D_h} \int_{-\infty}^{\infty} \mathrm{ReLU}[f_j]^2 Pr(f_j) df_j \qquad \qquad \text{From the definition of expectation.} \\ &= \sigma_\Omega^2 \sum_{j=1}^{D_h} \int_{-\infty}^{\infty} (\mathbb{I}[f_j > 0] f_j)^2 Pr(f_j) df_j \qquad \qquad \qquad \text{Only positive integral limits because of ReLU} \\ &= \sigma_\Omega^2 \sum_{j=1}^{D_h} \int_0^{\infty} f_j^2 Pr(f_j) df_j \qquad \qquad \qquad \text{Only positive integral limits because of ReLU} \\ &= \sigma_\Omega^2 \sum_{j=1}^{D_h} \frac{\sigma_f^2}{2} = \frac{D_h \sigma_\Omega^2 \sigma_f^2}{2} \qquad \qquad \qquad \text{Y2 of the variance for zero mean distribution} \end{split}$$

Since:

$$\sigma_{f'}^2 = \frac{D_h \sigma_{\Omega}^2 \sigma_f^2}{2}$$

Should choose:

$$\sigma_{\Omega}^2 = \frac{2}{D_h}$$

To get:

$$\sigma_{f'}^2 = \sigma_f^2$$

This is called He initialization or Kaiming initialization.

K. He, X. Zhang, S. Ren, and J. Sun, "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification," Proc. IEEE International Conference on Computer Vision, 2015, pp. 1026–1034. Accessed: Feb. 11, 2024.

He initialization (assumes ReLU)

• Forward pass: want the variance of hidden unit activations in layer k+1 to be the same as variance of activations in layer k:

$$\sigma_{\Omega}^2 = rac{2}{D_h}$$
 Number of units at layer k

• Backward pass: want the variance of gradients at layer k to be the same as variance of gradient in layer k+1:

$$\sigma_{\Omega}^2 = rac{2}{D_{h'}}$$
 Number of units at layer k+1

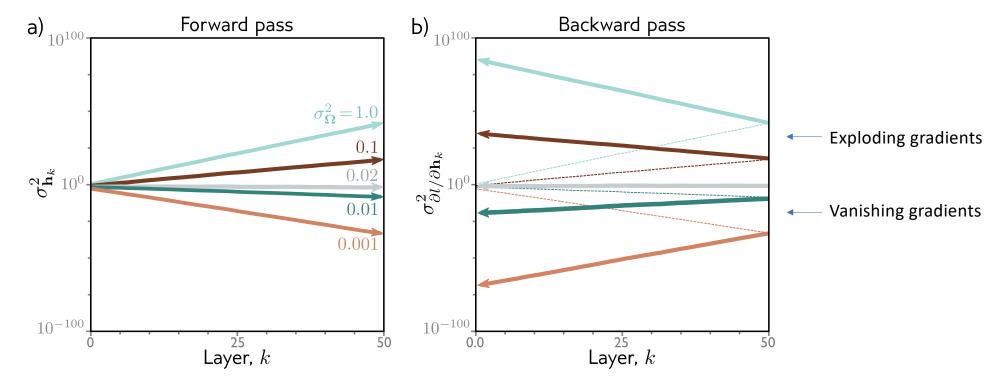


Figure 7.4 Weight initialization. Consider a deep network with 50 hidden layers and $D_h = 100$ hidden units per layer. The network has a 100 dimensional input \mathbf{x} initialized with values from a standard normal distribution, a single output fixed at y = 0, and a least squares loss function. The bias vectors $\boldsymbol{\beta}_k$ are initialized to zero and the weight matrices Ω_k are initialized with a normal distribution with mean zero and five different variances $\sigma_{\Omega}^2 \in \{0.001, 0.01, 0.02, 0.1, 1.0\}$. a)

$$\sigma_{\Omega}^2 = \frac{2}{D_h} = \frac{2}{100} = 0.02$$

Default Initialization in PyTorch

https://pytorch.org/docs/stable/nn.init.html#torch.nn.init.kaiming_uniform

torch.nn.init.kaiming_uniform_(tensor, a=0, mode='fan_in', nonlinearity='leaky_relu', generator=None) [SOURCE]

Fill the input Tensor with values using a Kaiming uniform distribution.

The method is described in *Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification* - He, K. et al. (2015). The resulting tensor will have values sampled from $\mathcal{U}(-\text{bound}, \text{bound})$ where

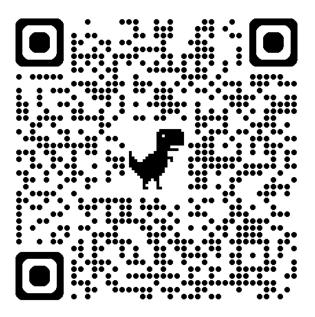
$$\mathrm{bound} = \mathrm{gain} \times \sqrt{\frac{3}{\mathrm{fan_mode}}}$$

Also known as He initialization.

Example Code

- Custom Code with Initialization Examples
- UDL Simple PyTorch Example

Feedback?



<u>Link</u>