BOSTON UNIVERSITY

Convolutional Networks

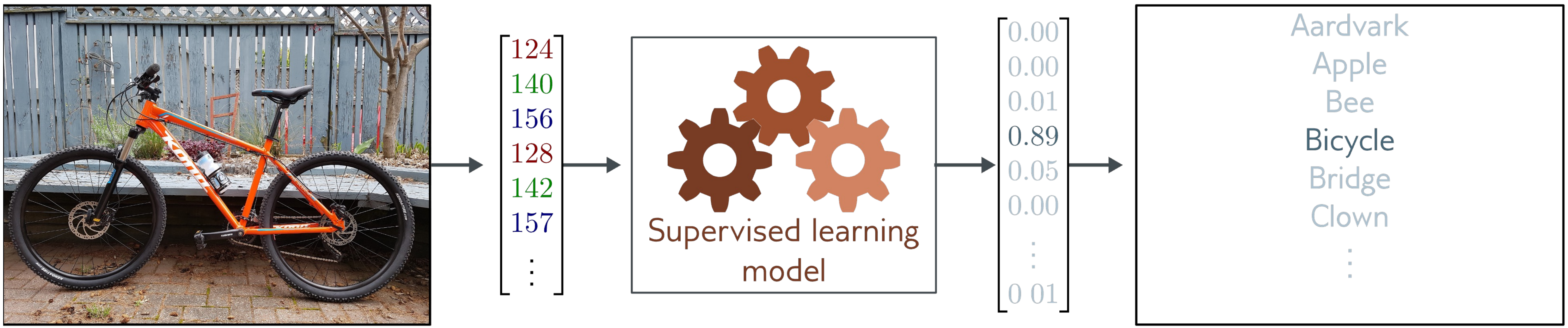
DL4DS – Spring 2024

Convolutional networks

- Networks for images
- Invariance and equivariance
- 1D convolution
- Convolutional layers
- Channels
- Receptive fields
- Convolutional network for MNIST 1D

Image classification

Real world input | Model input | Model | Model output | Real world output



- Multiclass classification problem (discrete classes, >2 possible classes)
- Convolutional network

Object detection

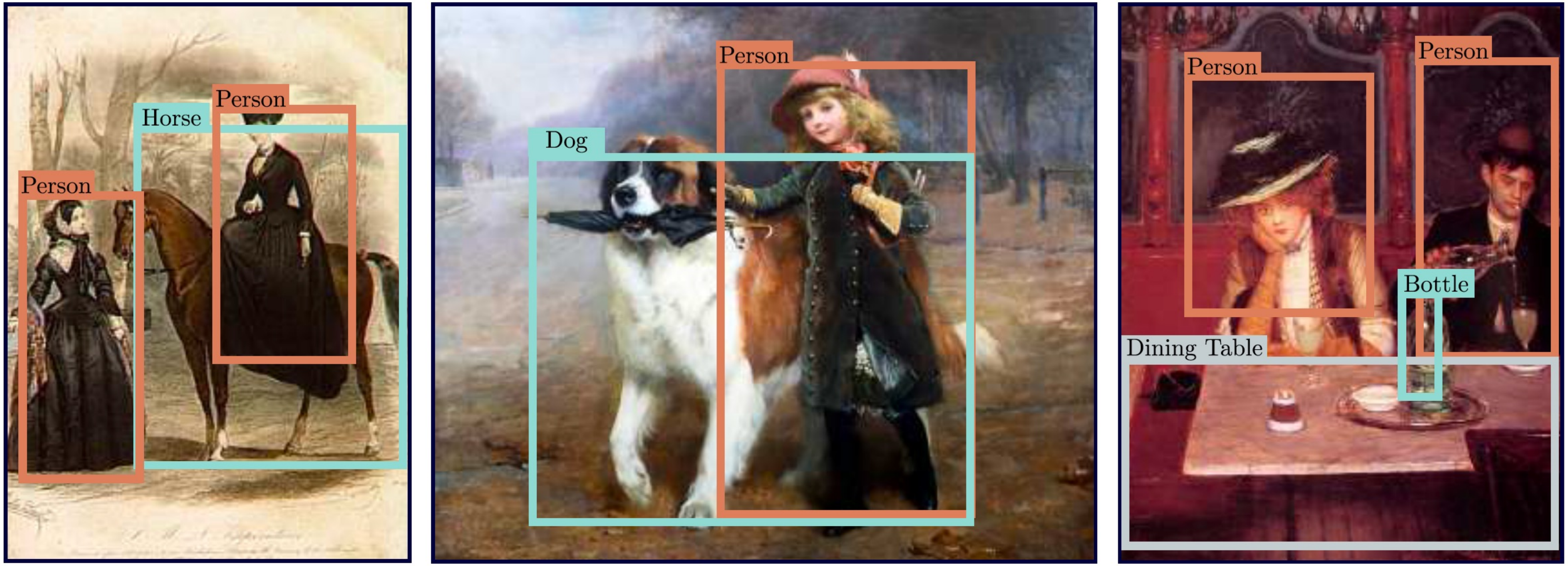
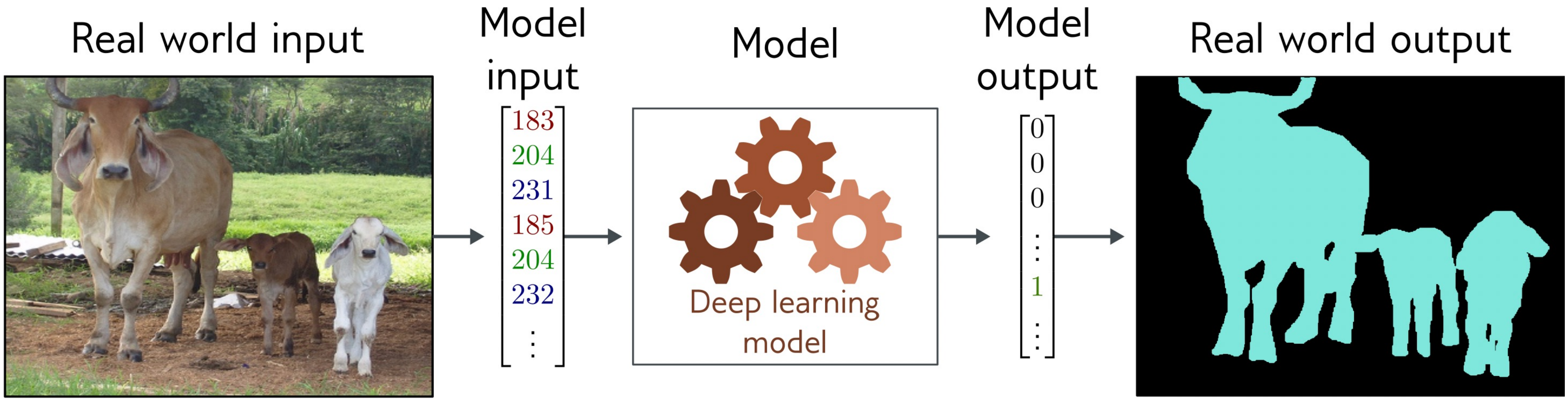


Image segmentation



- Multivariate binary classification problem (many outputs, two discrete classes)
- Convolutional encoder-decoder network

Networks for images

- Problems with fully-connected networks

1. Size
   - 224x224 RGB image = 150,528 dimensions
   - Hidden layers generally larger than inputs
   - One hidden layer = 150,520x150,528 weights -- 22 billion
2. Nearby pixels statistically related
   - But could permute pixels and relearn and get same results with FC
3. Should be stable under transformations
   - Don’t want to re-learn appearance at different parts of image

Convolutional networks

- Parameters only look at local image patches
- Share parameters across image

Convolutional networks

- Networks for images
- Invariance and equivariance
- 1D convolution
- Convolutional layers
- Channels
- Receptive fields
- Convolutional network for MNIST 1D

Invariance

- A function f[x] is invariant to a transformation t[] if:

$$\mathbf{f}\left[\mathbf{t}[\mathbf{x}]\right] = \mathbf{f}[\mathbf{x}]$$

i.e., the function output is the same even after the transformation is applied.

Invariance example

e.g., Image classification

- Image has been translated, but we want our classifier to give the same result

Equivariance

- A function f[x] is equivariant to a transformation t[] if:

$$\mathbf{f}\left[\mathbf{t}[\mathbf{x}]\right] = \mathbf{t}\left[\mathbf{f}[\mathbf{x}]\right]$$

i.e., the output is transformed in the same way as the input

Equivariance example

e.g., Image segmentation

- Image has been translated and we want segmentation to translate with it

Convolutional networks

- Networks for images
- Invariance and equivariance
- 1D convolution
- Convolutional layers
- Channels
- Receptive fields
- Convolutional network for MNIST 1D

Convolution* in 1D

- Input vector **x:**

$$\mathbf{x} = [x_1, x_2, \ldots, x_I]$$

- Output is weighted sum of neighbors:

$$z_i = \omega_1 x_{i-1} + \omega_2 x_i + \omega_3 x_{i+1}$$

- Convolutional kernel or filter:

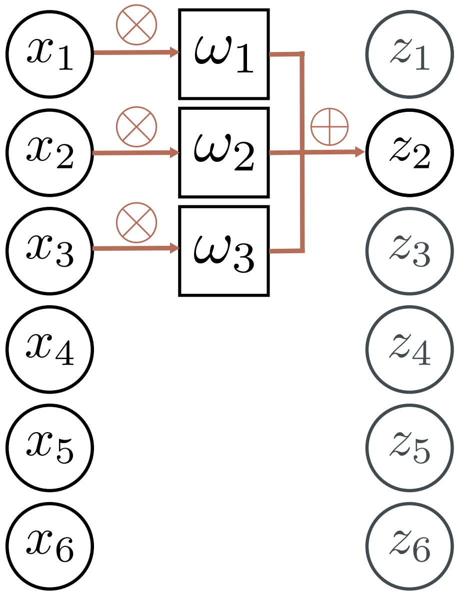
$$\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T$$

Kernel size = 3

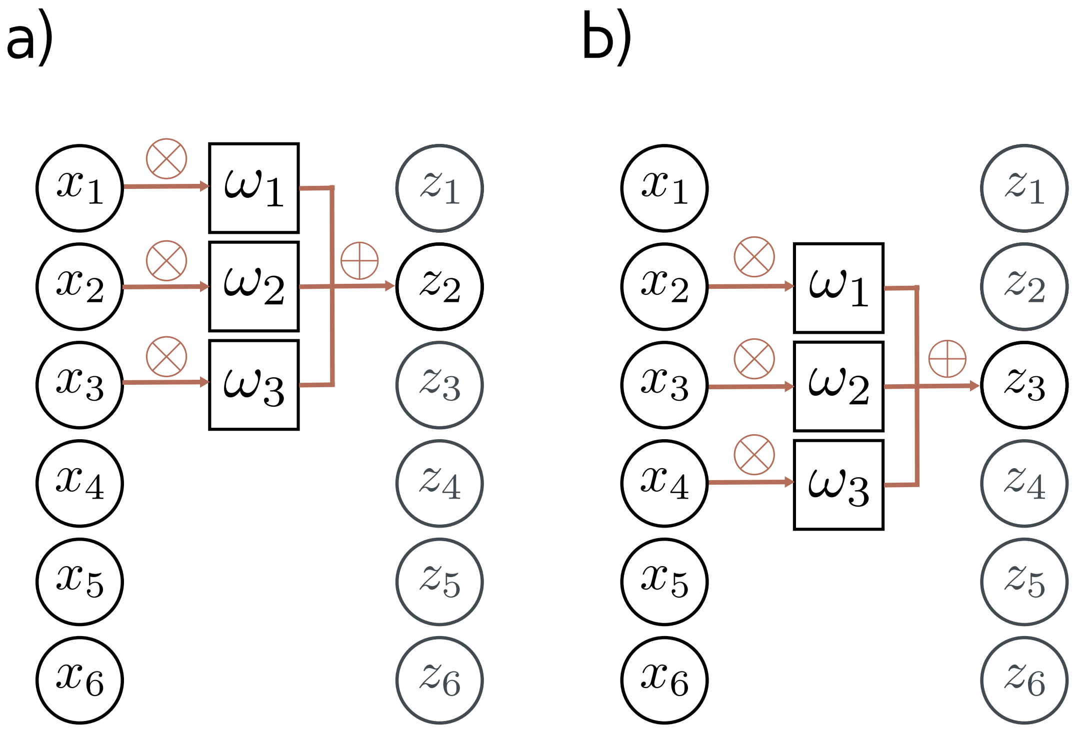
* Not really technically convolution

Convolution with kernel size 3

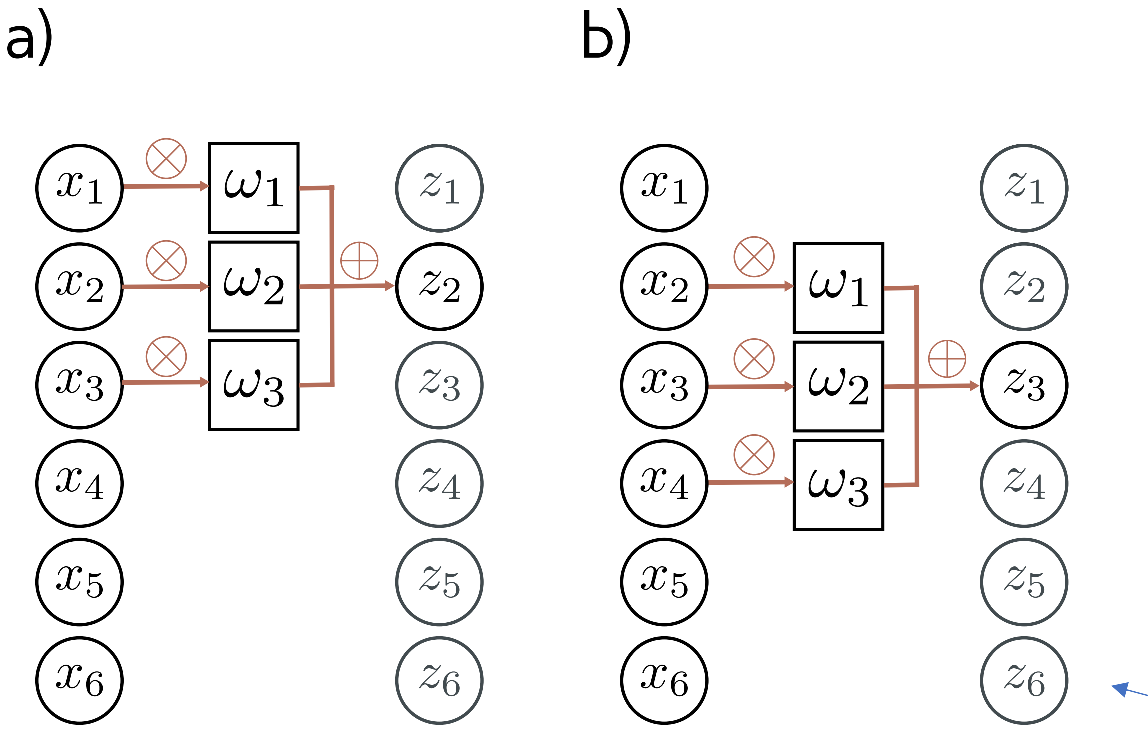
a)



Convolution with kernel size 3



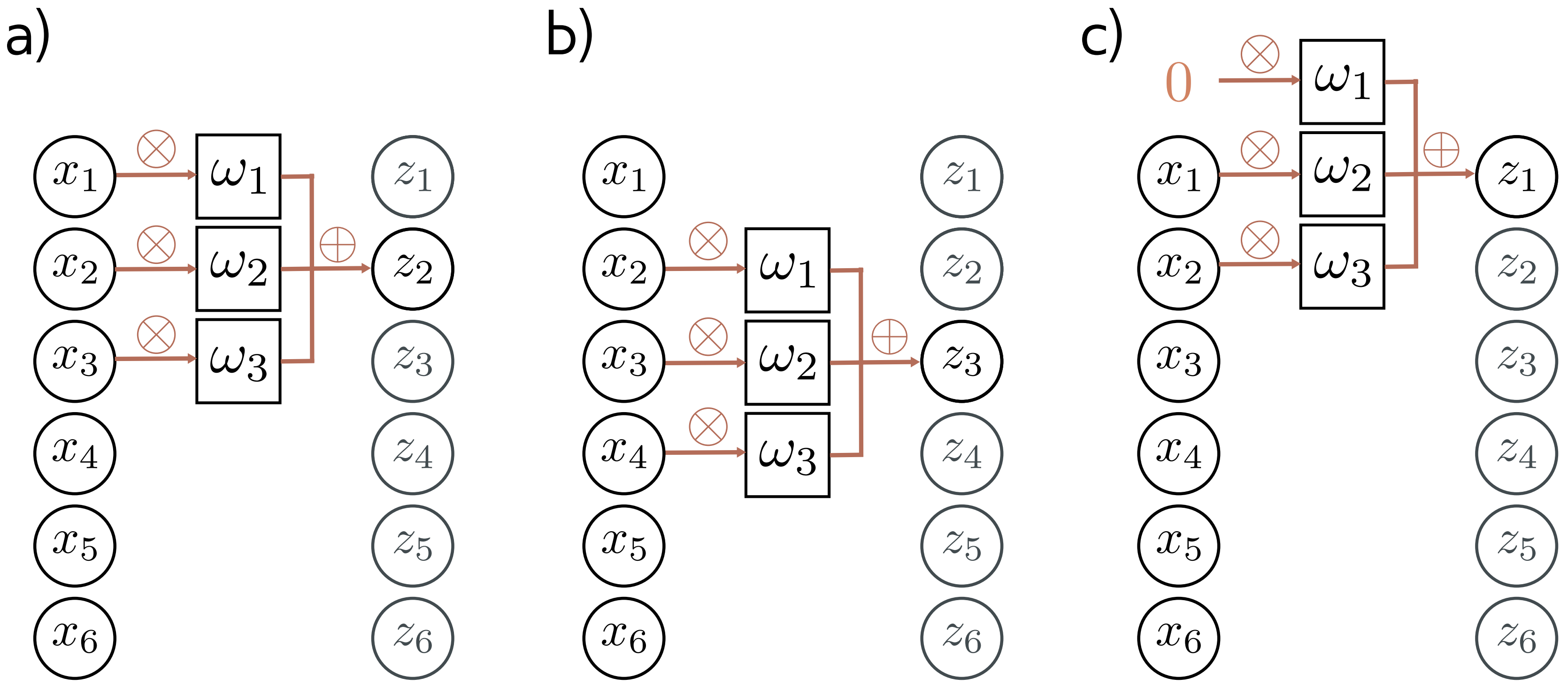
Convolution with kernel size 3



Equivariant to translation of input

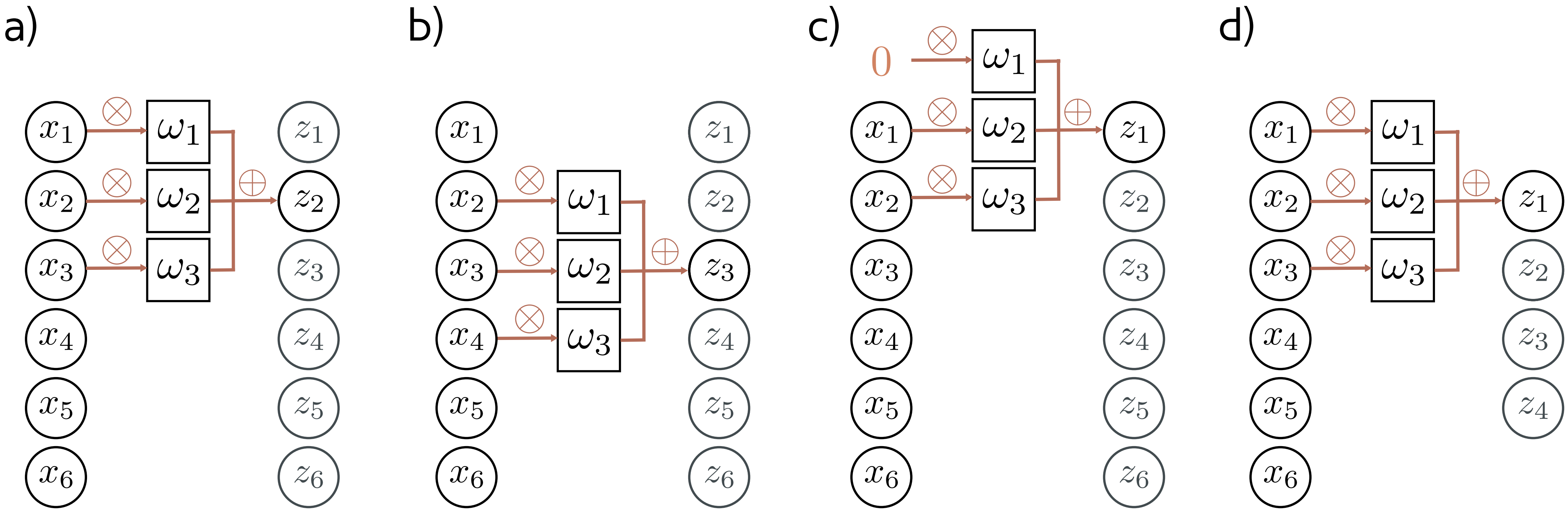
$$\mathbf{f}[\mathbf{t}[\mathbf{x}]] = \mathbf{t}[\mathbf{f}[\mathbf{x}]]$$

Zero padding



Treat positions that are beyond end of the input as zero.

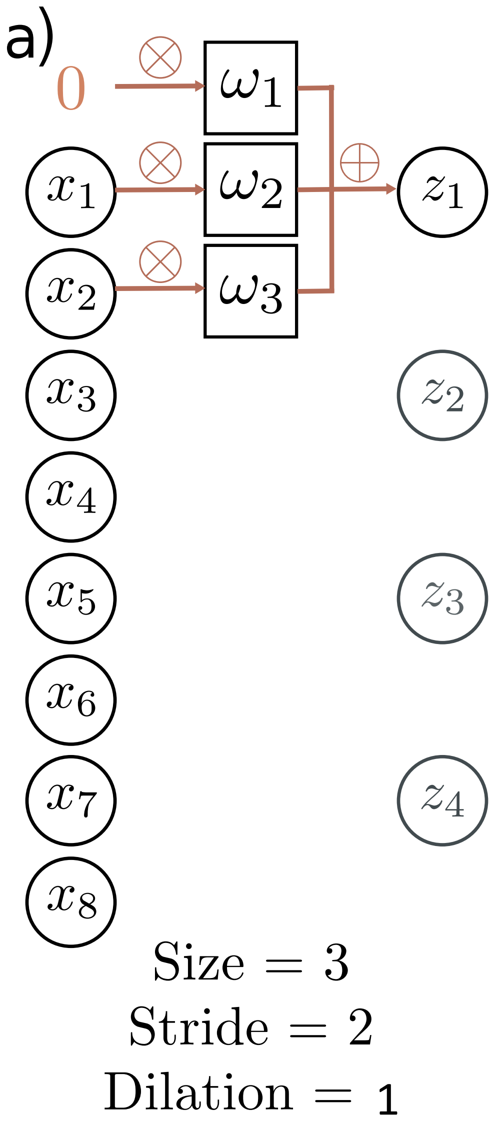
“Valid” convolutions

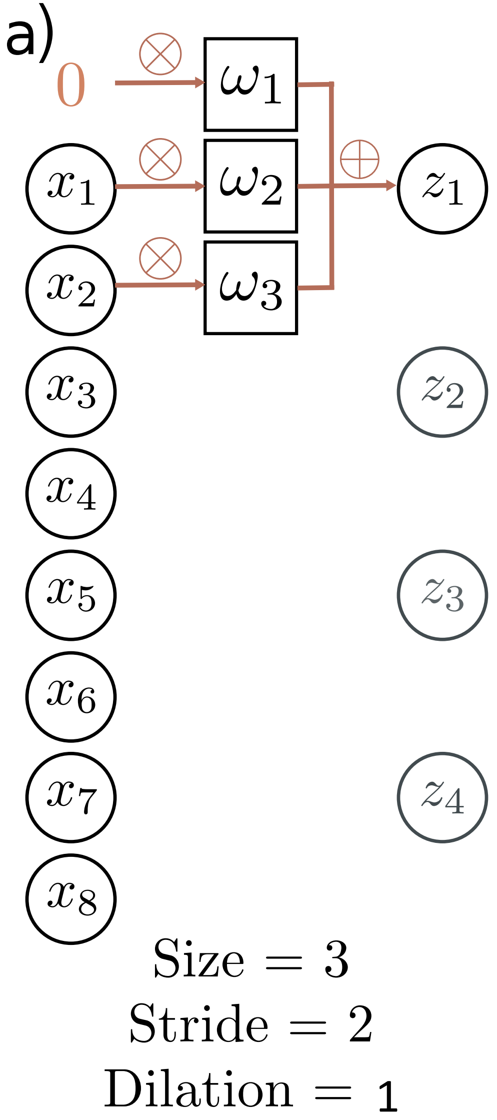


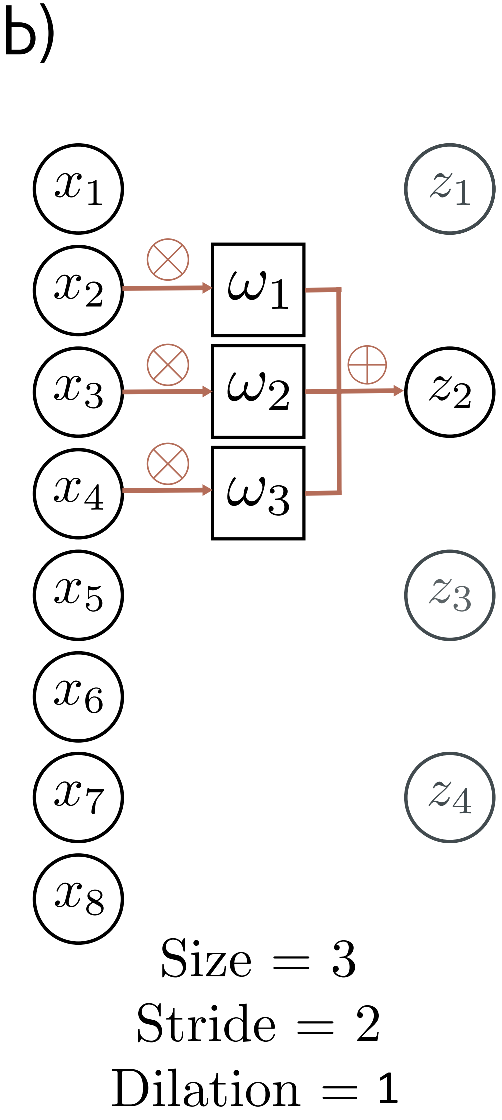
Only process positions where kernel falls in image (smaller output).

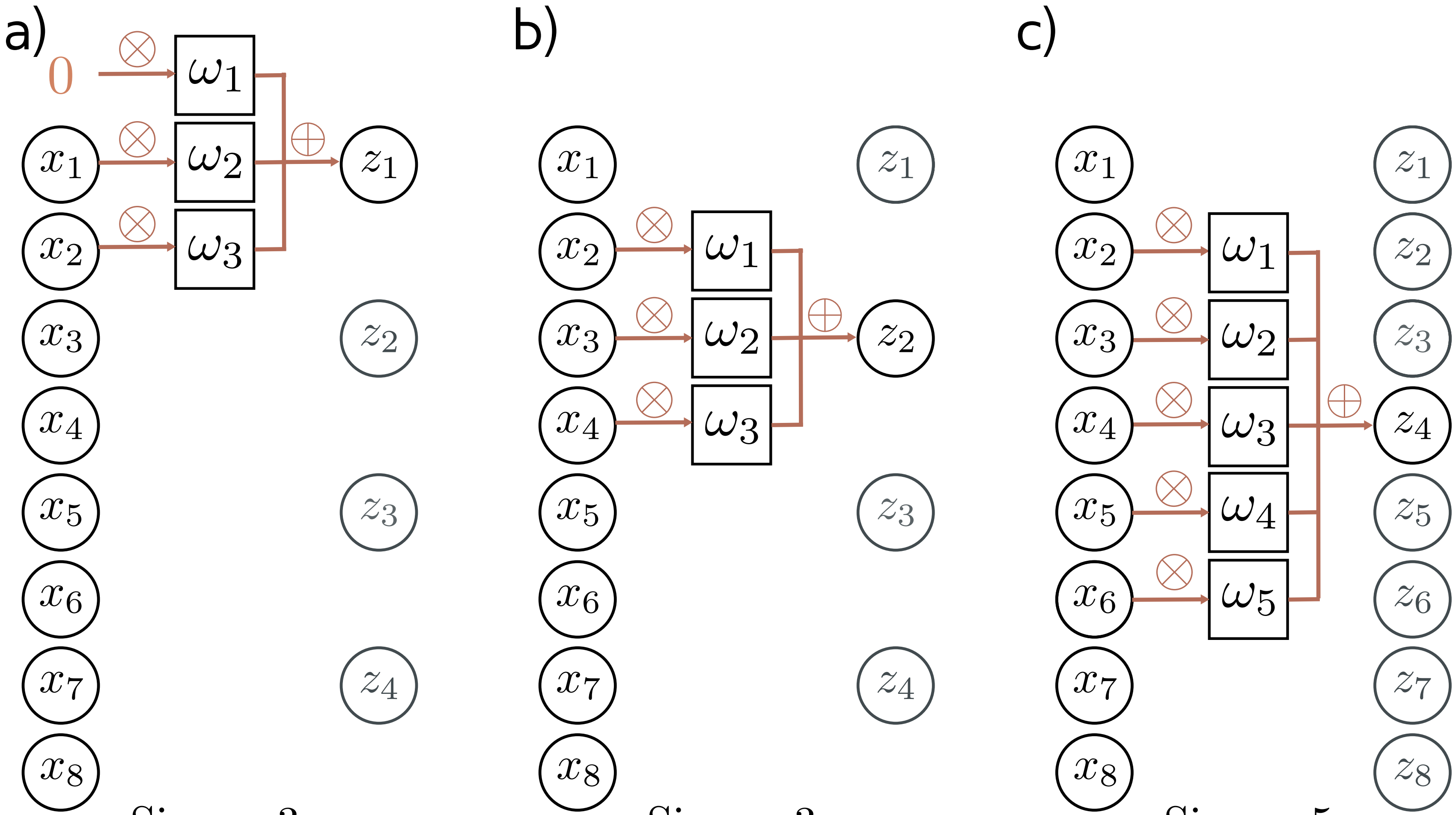
Stride, kernel size, and dilation

- Stride = shift by k positions for each output
  - Decreases size of output relative to input
- Kernel size = weight a different number of inputs for each output
  - Combine information from a larger area
  - But kernel size 5 uses 5 parameters
- Dilated or atrous convolutions = intersperse kernel values with zeros
  - Combine information from a larger area
  - Fewer parameters

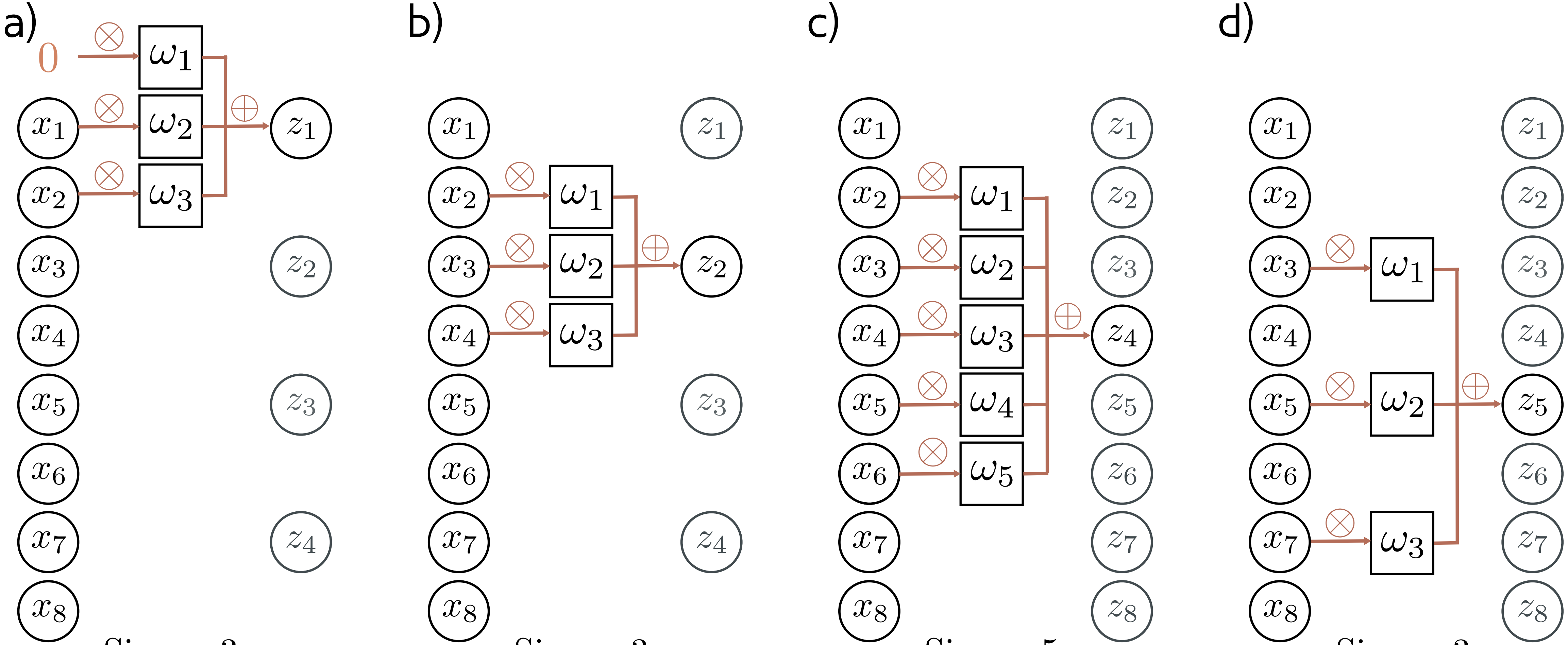

a)
0
ω_1
ω_2
ω_3
x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
z_1
z_2
z_3
z_4
Size = 3
Stride = 2
Dilation = 1


a)
0
ω_1
ω_2
ω_3
x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
z_1
z_2
z_3
z_4
Size $= 3$
Stride $= 2$
Dilation $= 1$


b)
x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
ω_1
ω_2
ω_3
z_1
z_2
z_3
z_4
Size $= 3$
Stride $= 2$
Dilation $= 1$



a)	b)	c)
Size $= 3$	Size $= 3$	Size $= 5$
Stride $= 2$	Stride $= 2$	Stride $= 1$
Dilation $= 1$	Dilation $= 1$	Dilation $= 1$



a) Size = 3
Stride = 2
Dilation = 1

b) Size = 3
Stride = 2
Dilation = 1

c) Size = 5
Stride = 1
Dilation = 1

d) Size = 3
Stride = 1
Dilation = 2

Convolutional networks

- Networks for images
- Invariance and equivariance
- 1D convolution
- Convolutional layers
- Channels
- Receptive fields
- Convolutional network for MNIST 1D

Convolutional layer

$$
\begin{aligned}
h_i &= \mathrm{a}\left[\beta + \omega_1 x_{i-1} + \omega_2 x_i + \omega_3 x_{i+1}\right] \\
&= \mathrm{a}\left[\beta + \sum_{j=1}^{3} \omega_j x_{i+j-2}\right]
\end{aligned}
$$

Special case of fully-connected network

Convolutional network:

$$
\begin{aligned}
h_i &= \mathrm{a}\left[\beta + \omega_1 x_{i-1} + \omega_2 x_i + \omega_3 x_{i+1}\right] \\
&= \mathrm{a}\left[\beta + \sum_{j=1}^{3} \omega_j x_{i+j-2}\right]
\end{aligned}
$$

Fully connected network:

$$
h_i = \mathrm{a}\left[\beta_i + \sum_{j=1}^{D} \omega_{ij} x_j\right]
$$

Special case of fully-connected network

Convolutional network:

$$h_i = \mathrm{a}\left[\beta + \omega_1 x_{i-1} + \omega_2 x_i + \omega_3 x_{i+1}\right]$$
$$= \mathrm{a}\left[\beta + \sum_{j=1}^{3} \omega_j x_{i+j-2}\right]$$

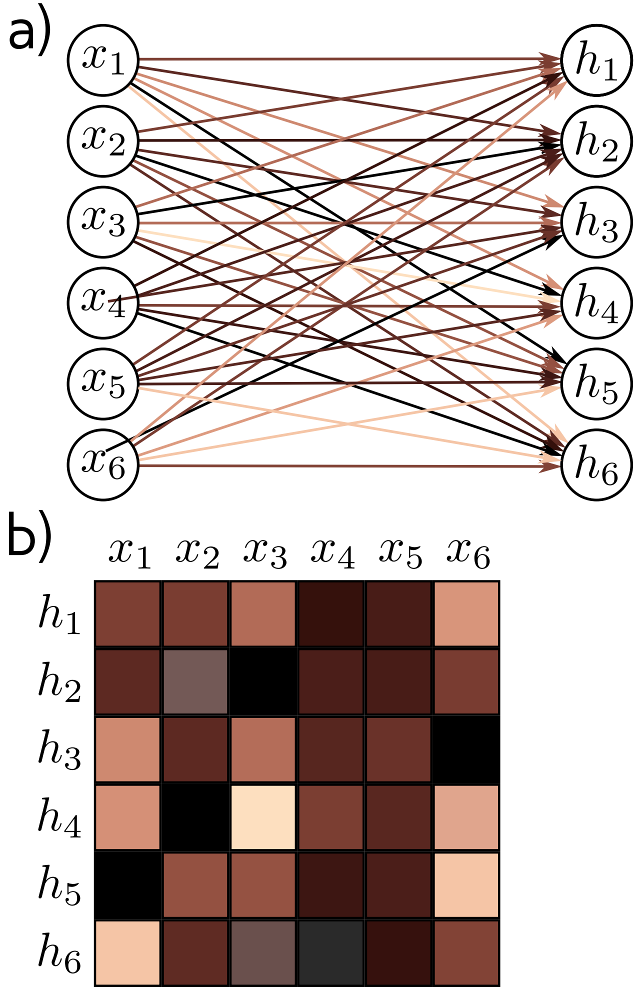
3 weights, 1 bias

Fully connected network:

$$h_i = \mathrm{a}\left[\beta_i + \sum_{j=1}^{D} \omega_{ij} x_j\right]$$

D^2 weights, D biases

Special case of fully-connected network

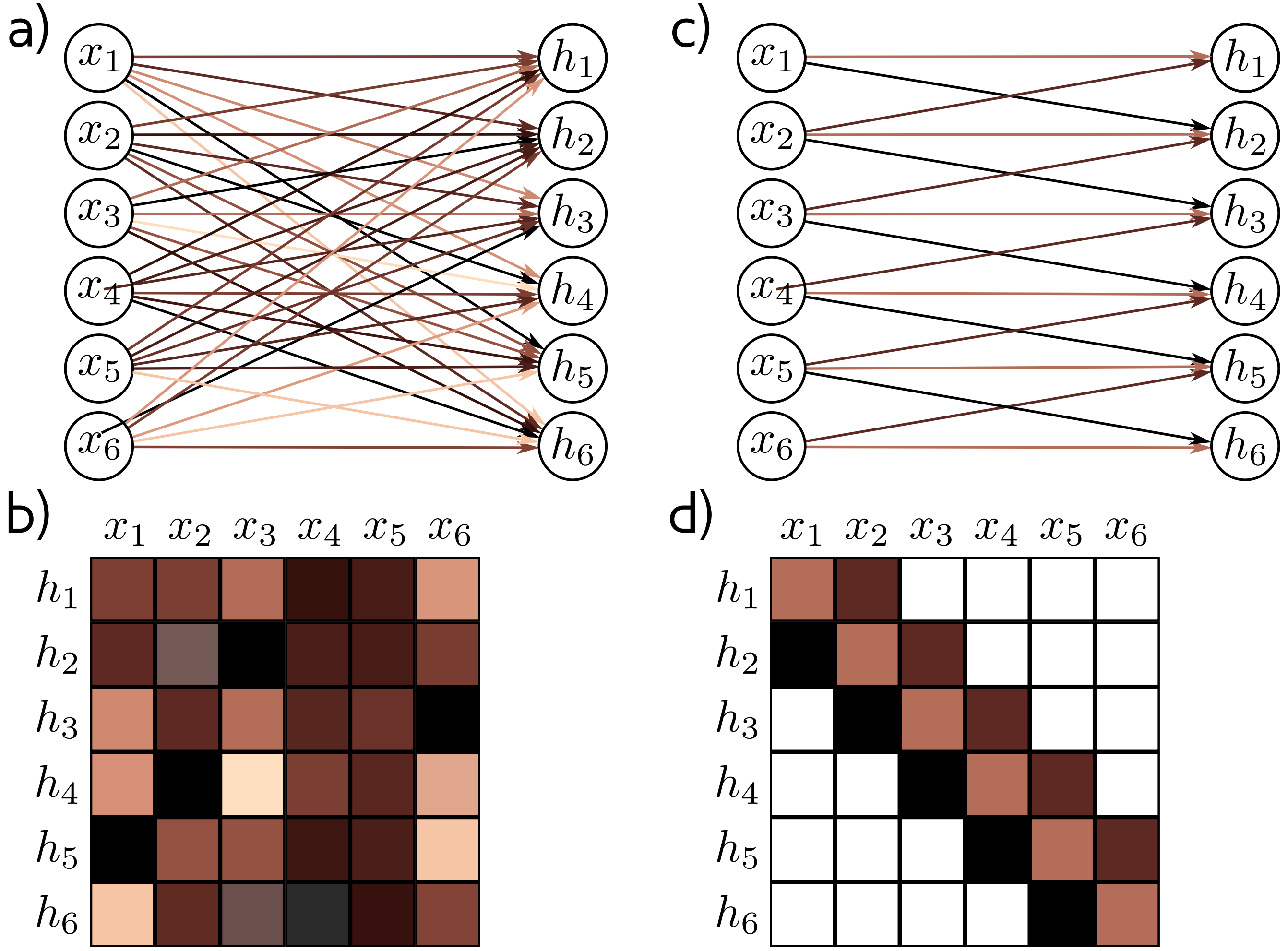


Weight Matrix

Fully connected network

Bias is implied

Special case of fully-connected network



Weight Matrices

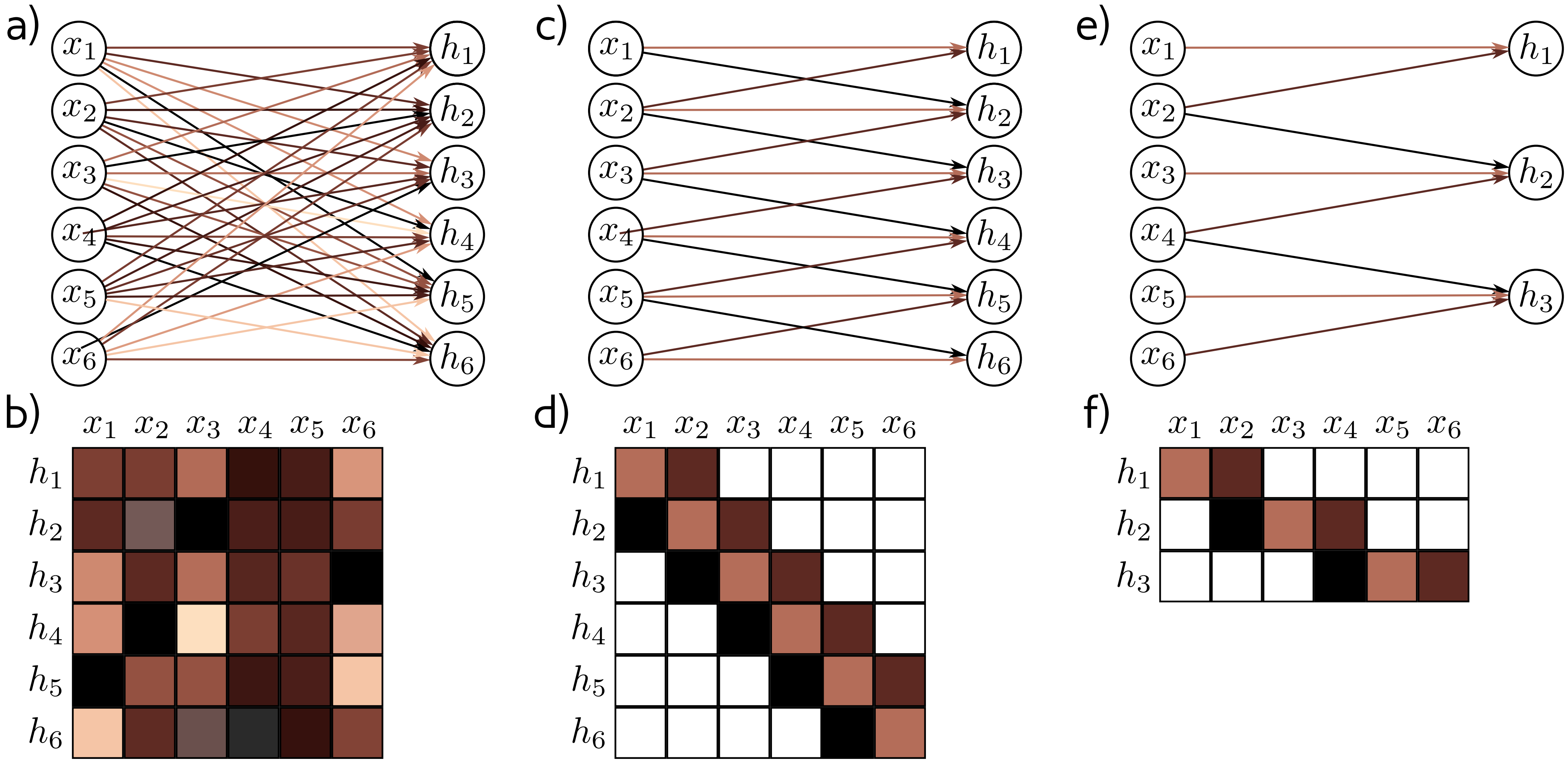
Fully connected network

Convolution, kernel 3, stride 1, dilation 1

Bias is implied

Special case of fully-connected network

Bias is implied



Weight Matrices

Fully connected network

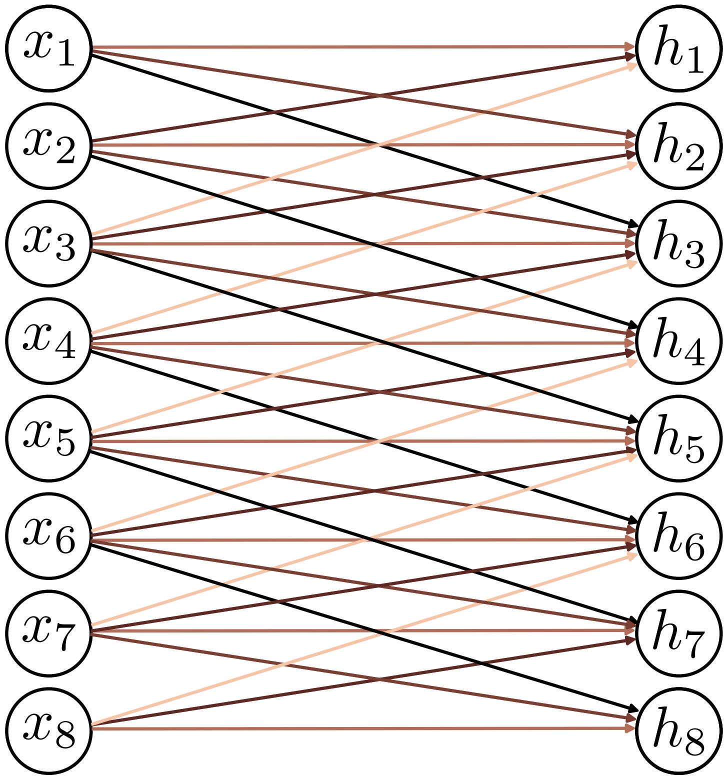
Convolution, size 3, stride 1, dilation 1, zero padding

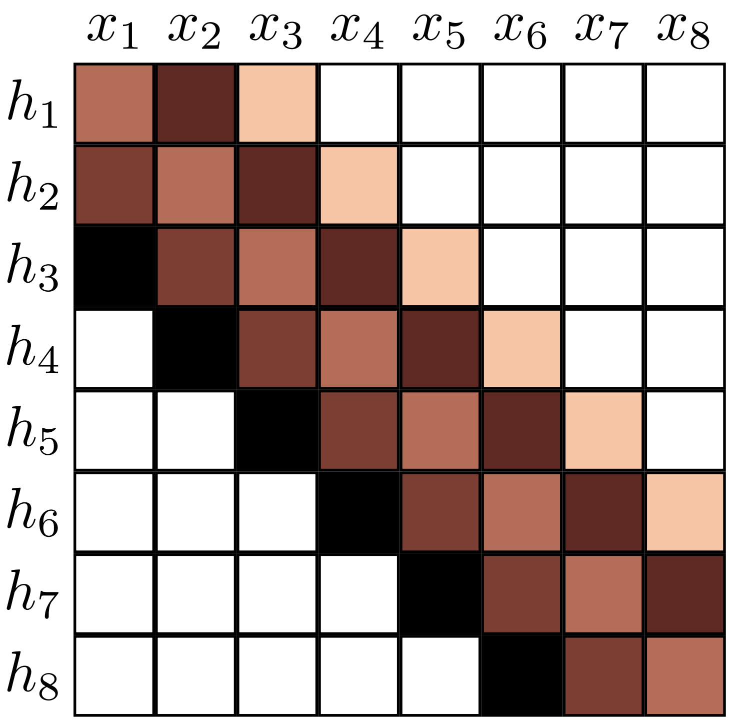
Convolution, size 3, stride 2, dilation 1, zero padding

Question 1

Bias is implied

- Kernel size?
- Stride?
- Dilation?
- Zero padding / valid?





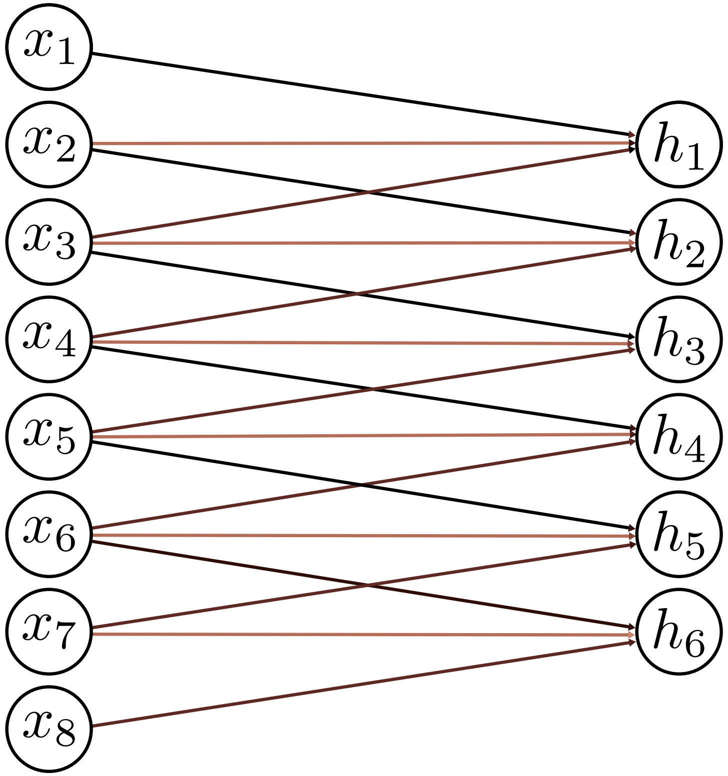
Convolution Configuration

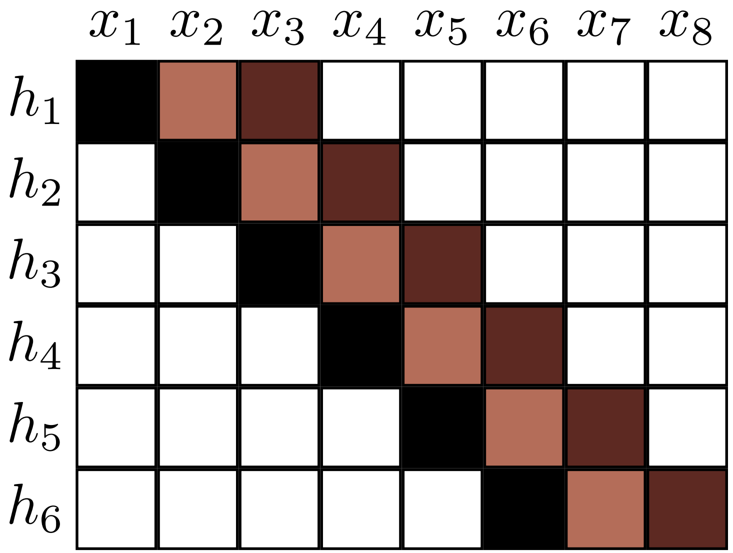
ⓘ Start presenting to display the poll results on this slide.

Question 2

Bias is implied

- Kernel size?
- Stride?
- Dilation?
- Zero padding / valid?





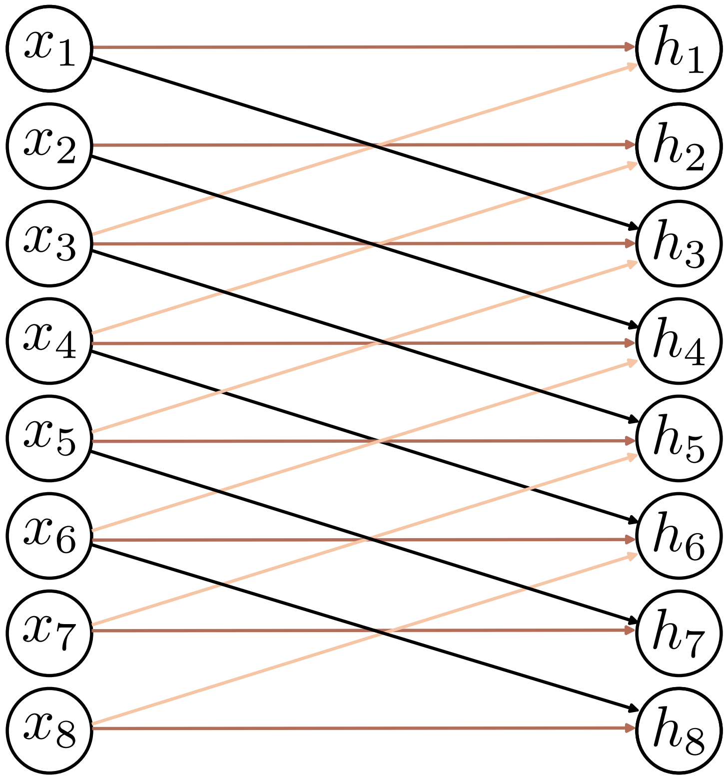
Conv Config 2

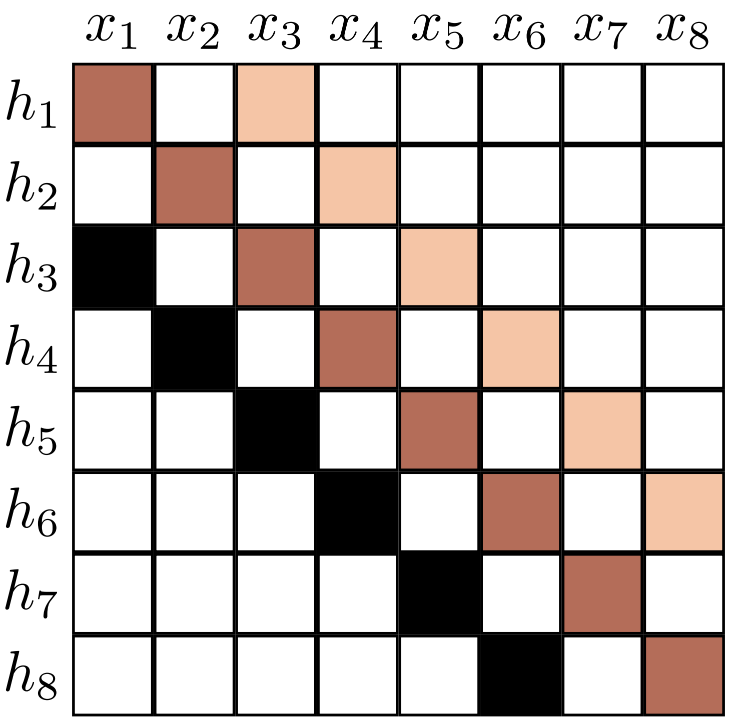
ⓘ Start presenting to display the poll results on this slide.

Question 3

Bias is implied

- Kernel size?
- Stride?
- Dilation?
- Zero padding / valid?





slido

Conv Config 3

ⓘ Start presenting to display the poll results on this slide.

Convolutional networks

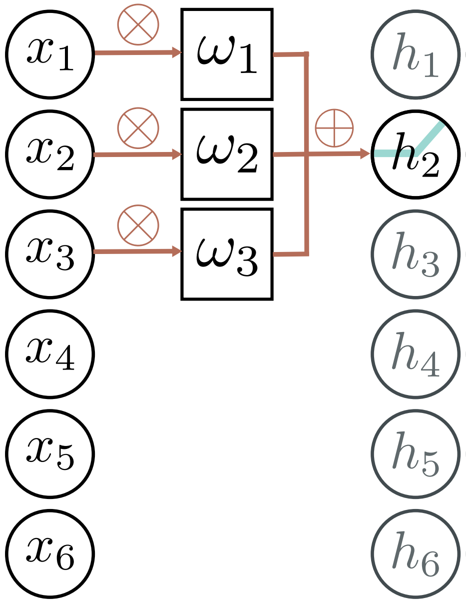
- Networks for images
- Invariance and equivariance
- 1D convolution
- Convolutional layers
- Channels
- Receptive fields
- Convolutional network for MNIST 1D

Channels

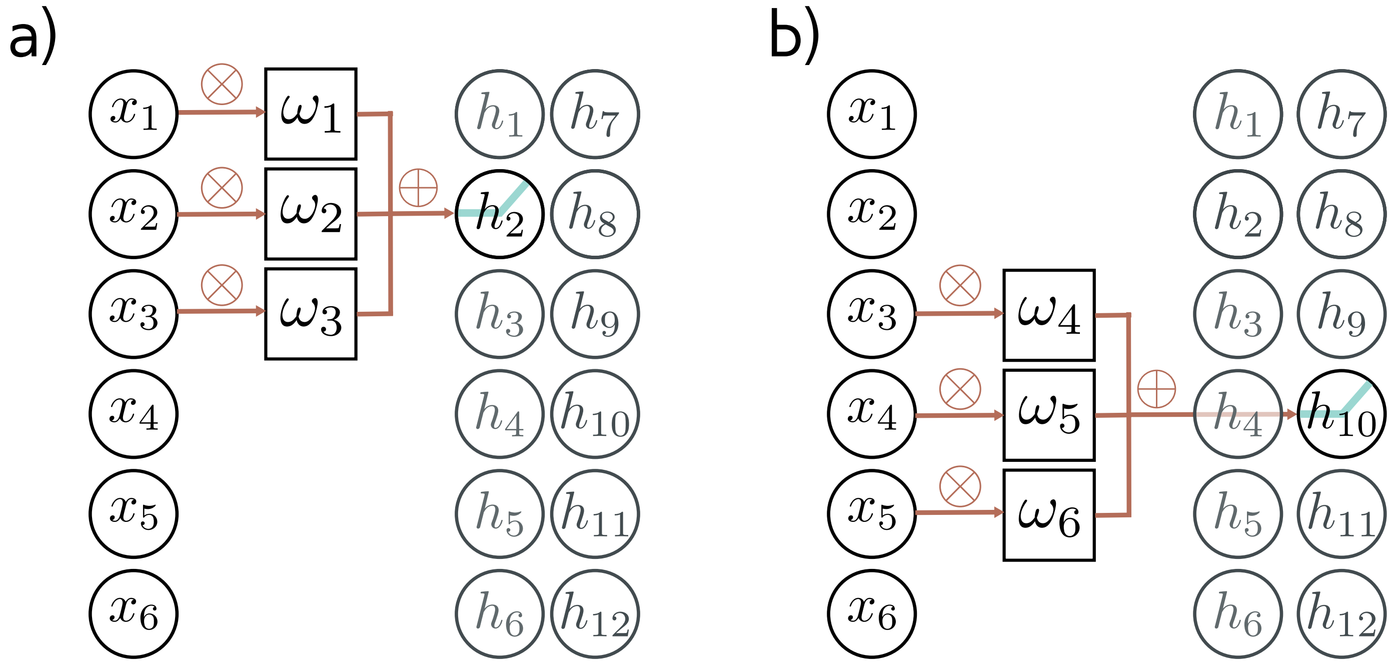
- The convolutional operation averages together the inputs
- Plus passes through ReLU function
- Result is loss of information
- Solution:
  - apply several convolutions and stack them in channels
  - Sometimes also called feature maps

Two output channels, one input channel

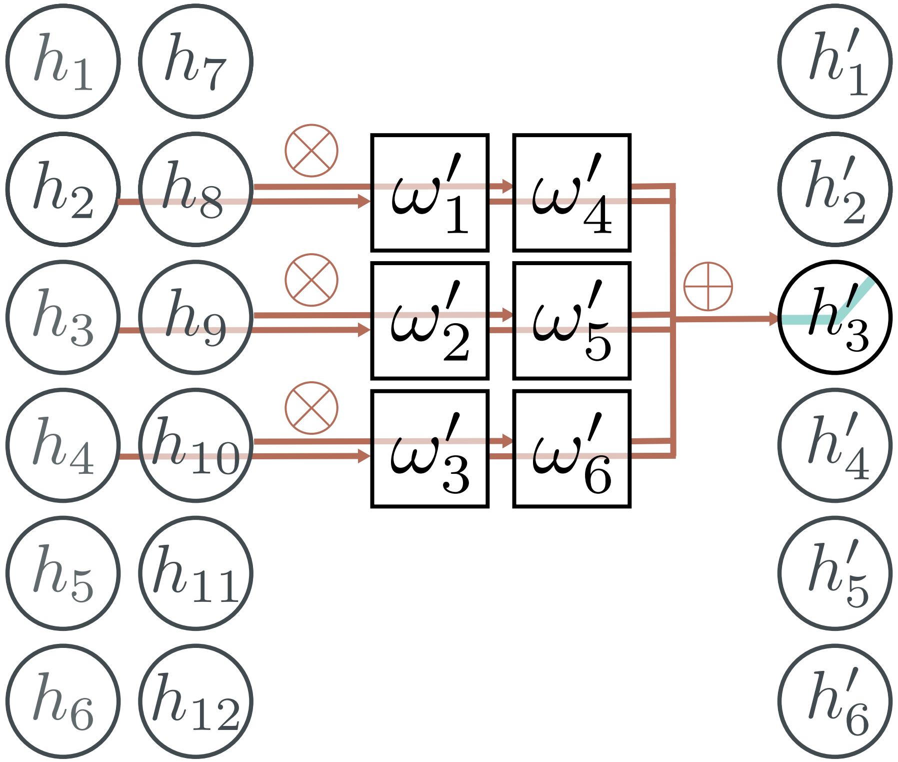
a)



Two output channels, one input channel



Two input channels, one output channel



How many parameters?

- If there are C_i input channels and kernel size K

$$\mathbf{\Omega} \in \mathbb{R}^{C_i \times K} \qquad \boldsymbol{\beta} \in \mathbb{R}$$

- If there are C_i input channels and C_o output channels

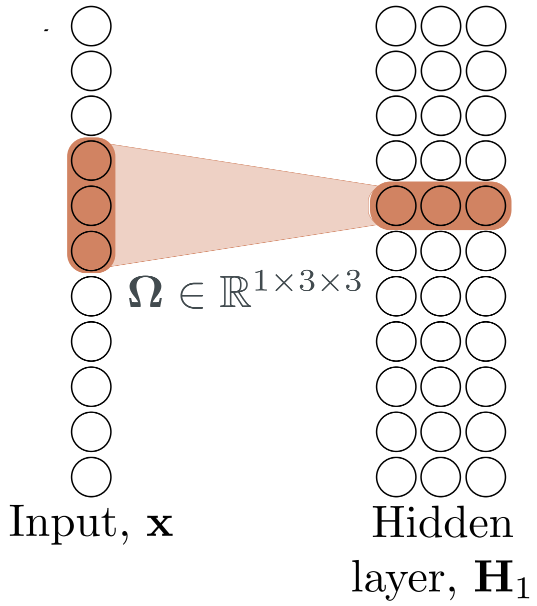
$$\mathbf{\Omega} \in \mathbb{R}^{C_i \times C_o \times K} \qquad \boldsymbol{\beta} \in \mathbb{R}^{C_o}$$

Convolutional networks

- Networks for images
- Invariance and equivariance
- 1D convolution
- Convolutional layers
- Channels
- Receptive fields
- Convolutional network for MNIST 1D

Receptive fields

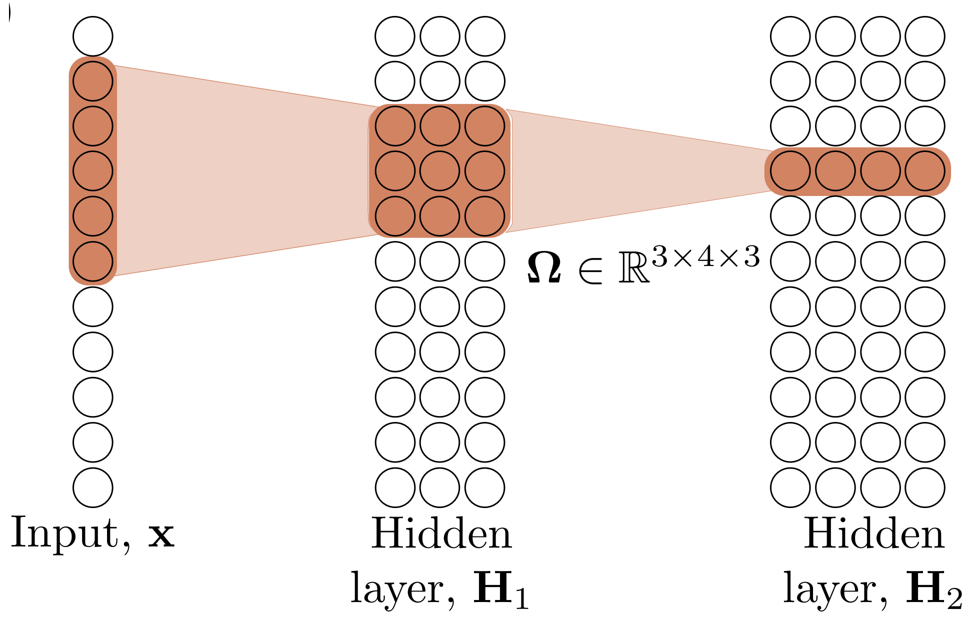
$\mathbb{R}^{C_i \times C_o \times K}$



Receptive fields

$\mathbb{R}^{C_i \times C_o \times K}$

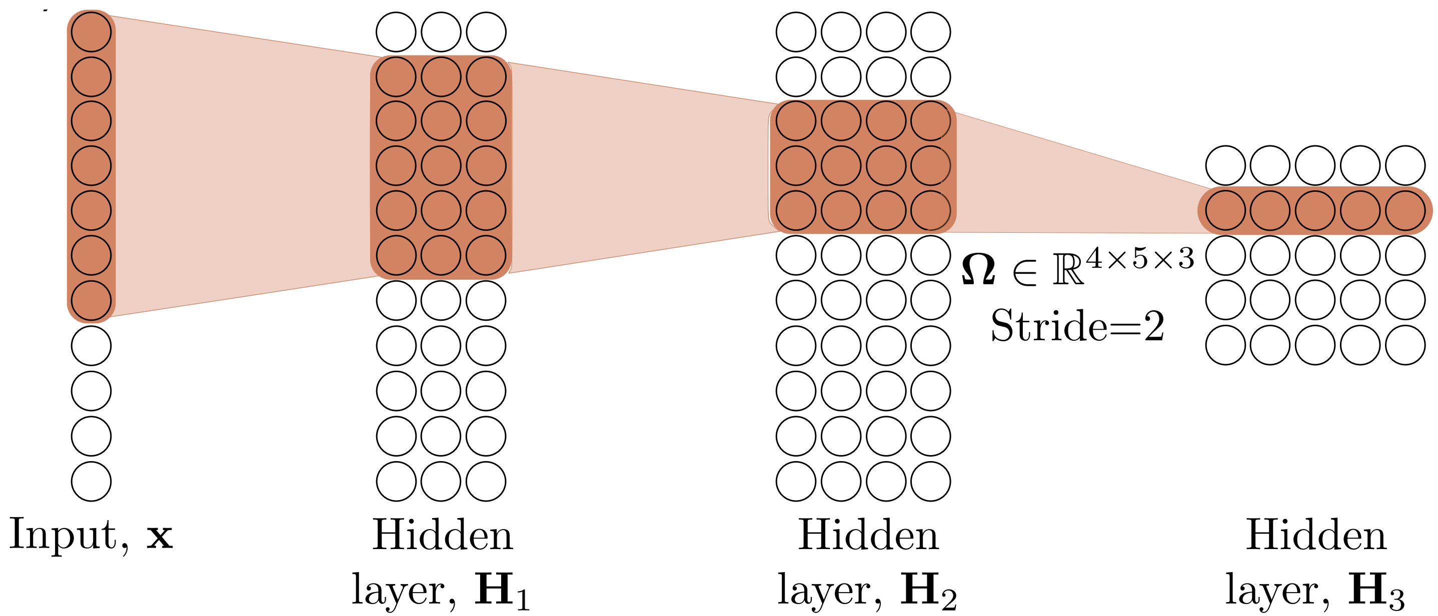
$$[\omega_1, \omega_2, \omega_3] \otimes [\omega_1, \omega_2, \omega_3] = [\omega_1, \omega_2, \omega_3, \omega_4, \omega_5]$$



Receptive fields

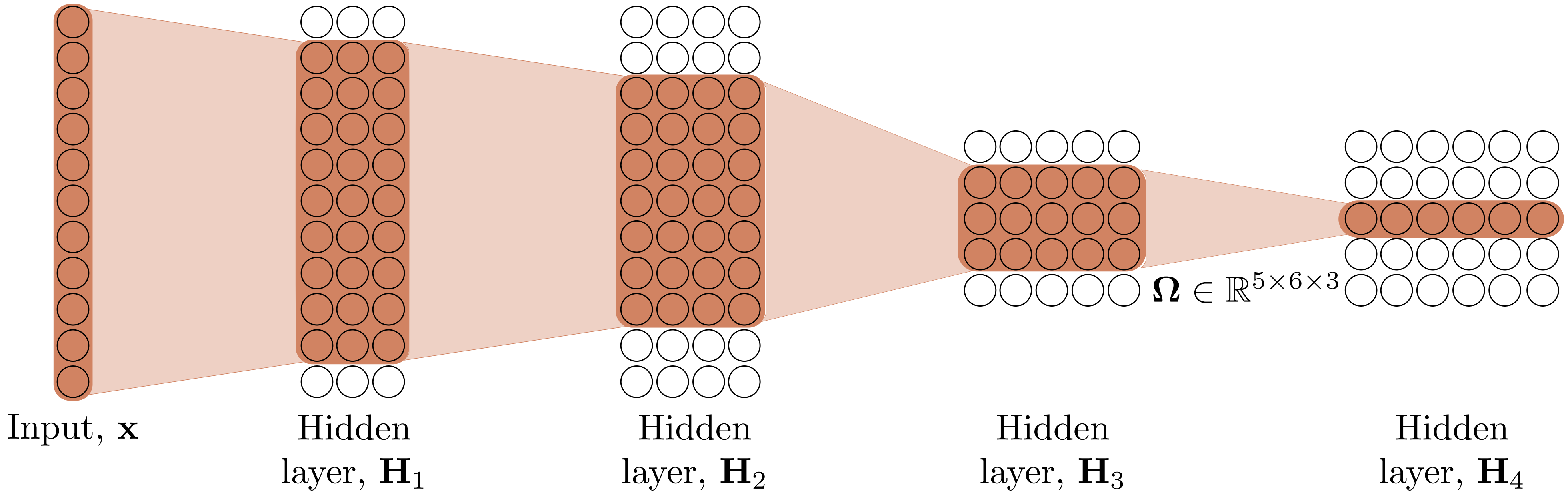
$\mathbb{R}^{C_i \times C_o \times K}$

$$[\omega_1, \omega_2, \omega_3, \omega_4, \omega_5] \otimes [\omega_1, \omega_2, \omega_3] = [\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7]$$



Receptive fields

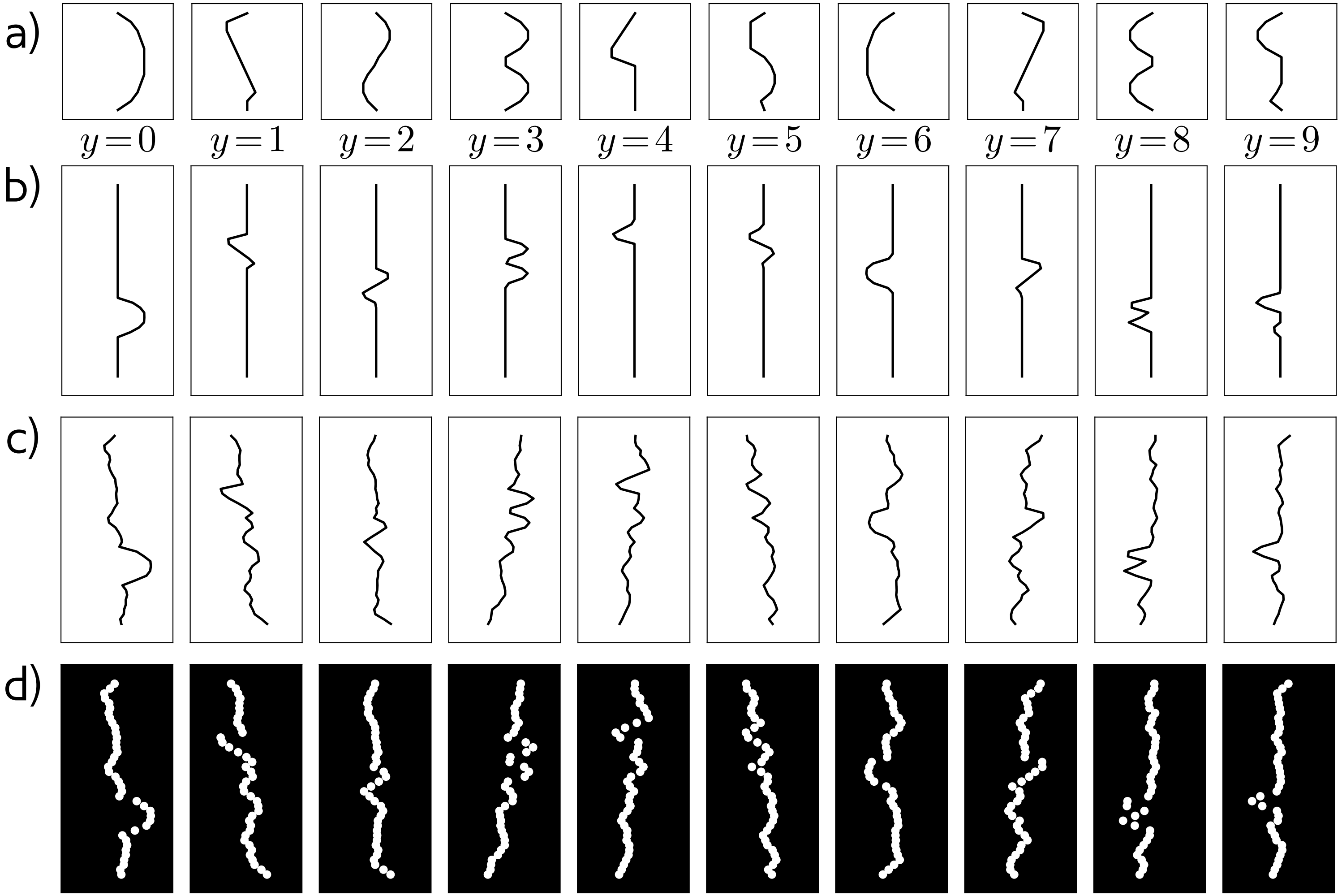
$\mathbb{R}^{C_i \times C_o \times K}$



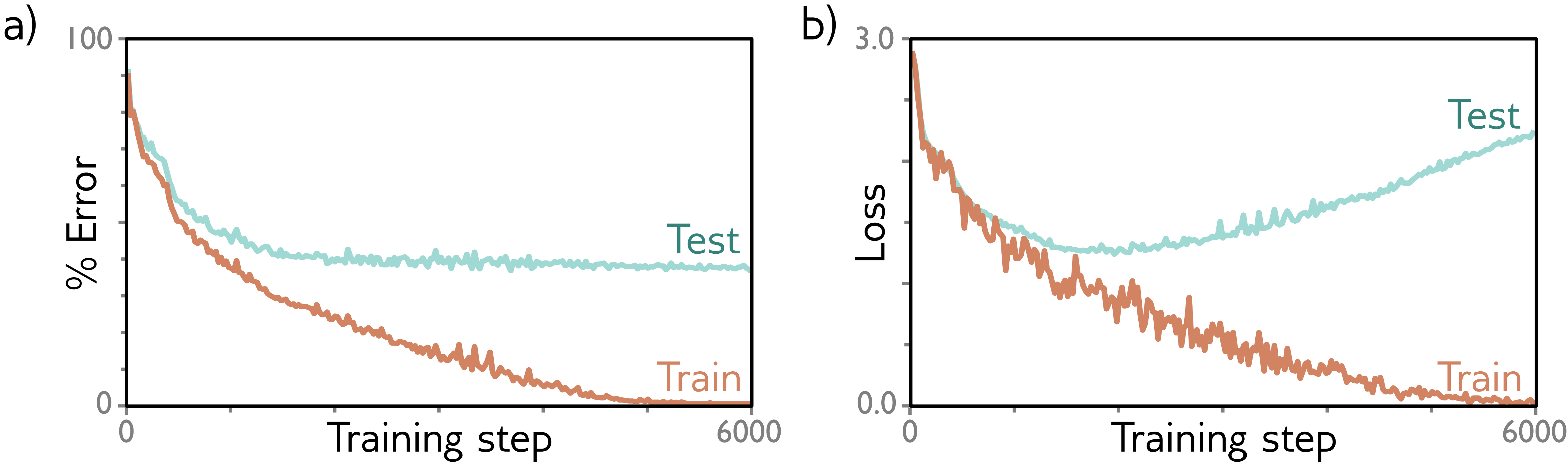
Convolutional networks

- Networks for images
- Invariance and equivariance
- 1D convolution
- Convolutional layers
- Channels
- Receptive fields
- Convolutional network for MNIST 1D

MNIST 1D Dataset



MNIST-1D results for fully-connected network



Fully connected network

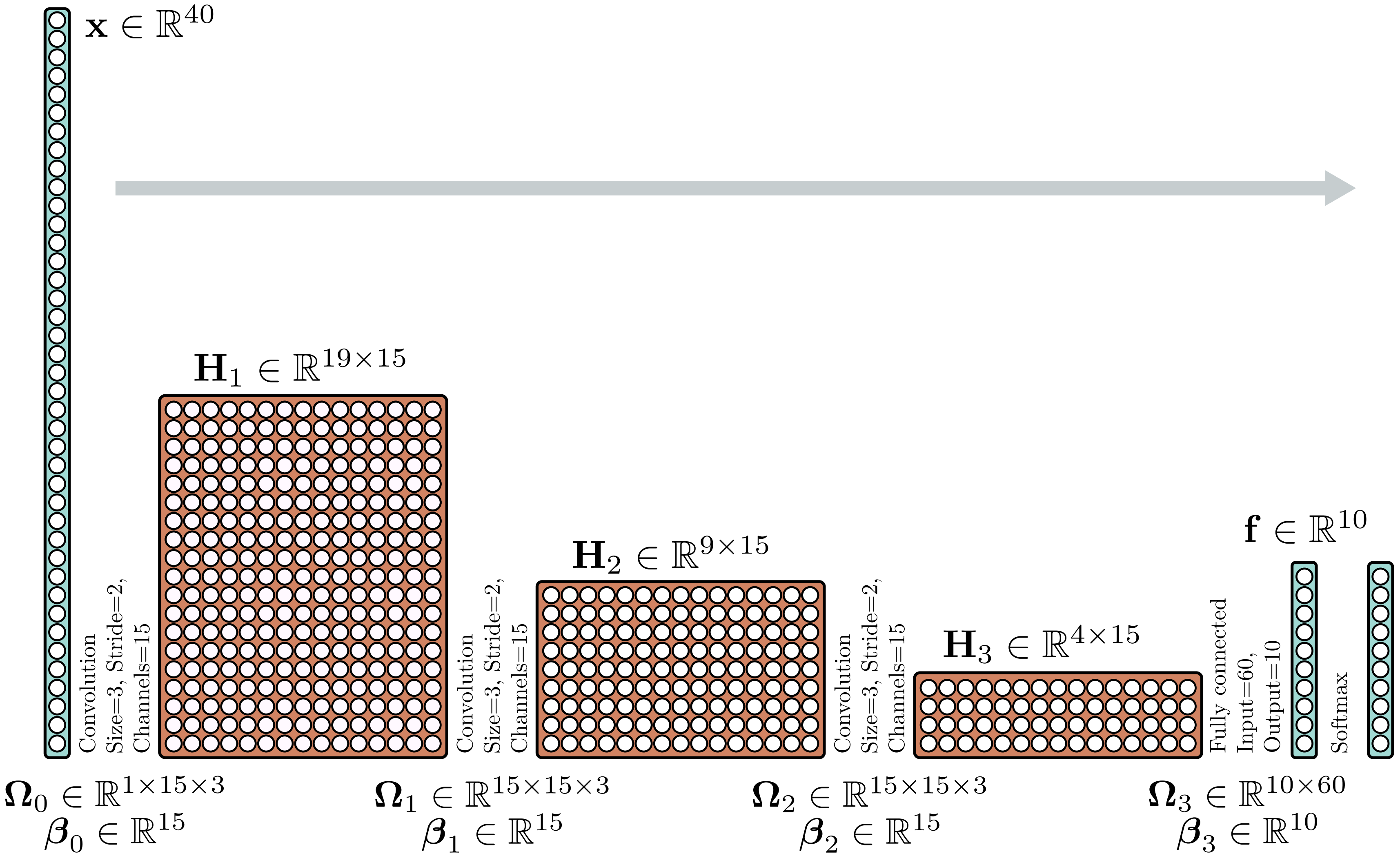
- Exactly same number of layers and hidden units
- All fully-connected layers
- Total parameters = 150,185

Convolutional network

- Four hidden layers
- Three convolutional layers
- One fully-connected layer
- Softmax at end
- Total parameters = 2050
- Trained for 100,000 steps with SGD, LR = 0.01, batch size 100

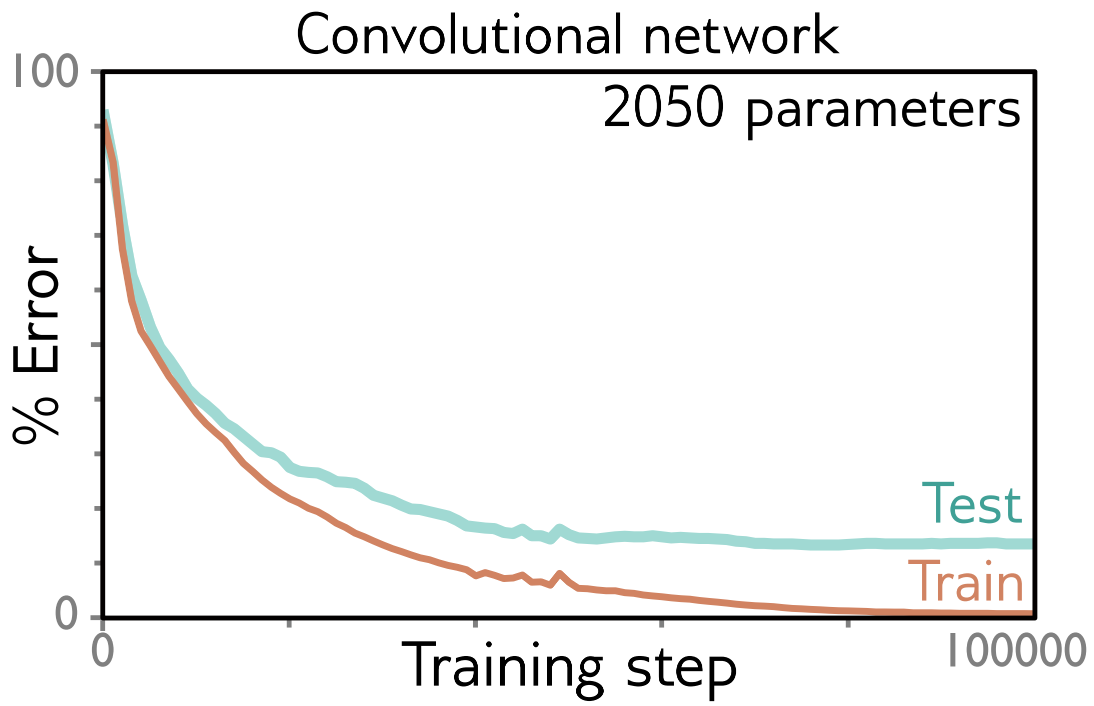
```
==========================================================================================
Layer (type:depth-idx)                   Output Shape              Param #
==========================================================================================
Sequential                               [100, 10]                 --
├─Conv1d: 1-1                            [100, 15, 19]             60
├─ReLU: 1-2                              [100, 15, 19]             --
├─Conv1d: 1-3                            [100, 15, 9]              690
├─ReLU: 1-4                              [100, 15, 9]              --
├─Conv1d: 1-5                            [100, 15, 4]              690
├─ReLU: 1-6                              [100, 15, 4]              --
├─Flatten: 1-7                           [100, 60]                 --
├─Linear: 1-8                            [100, 10]                 610
==========================================================================================
Total params: 2,050
Trainable params: 2,050
Non-trainable params: 0
Total mult-adds (Units.MEGABYTES): 1.07
==========================================================================================
Input size (MB): 0.02
Forward/backward pass size (MB): 0.39
Params size (MB): 0.01
Estimated Total Size (MB): 0.42
==========================================================================================
```

MNIST-1D convolutional network

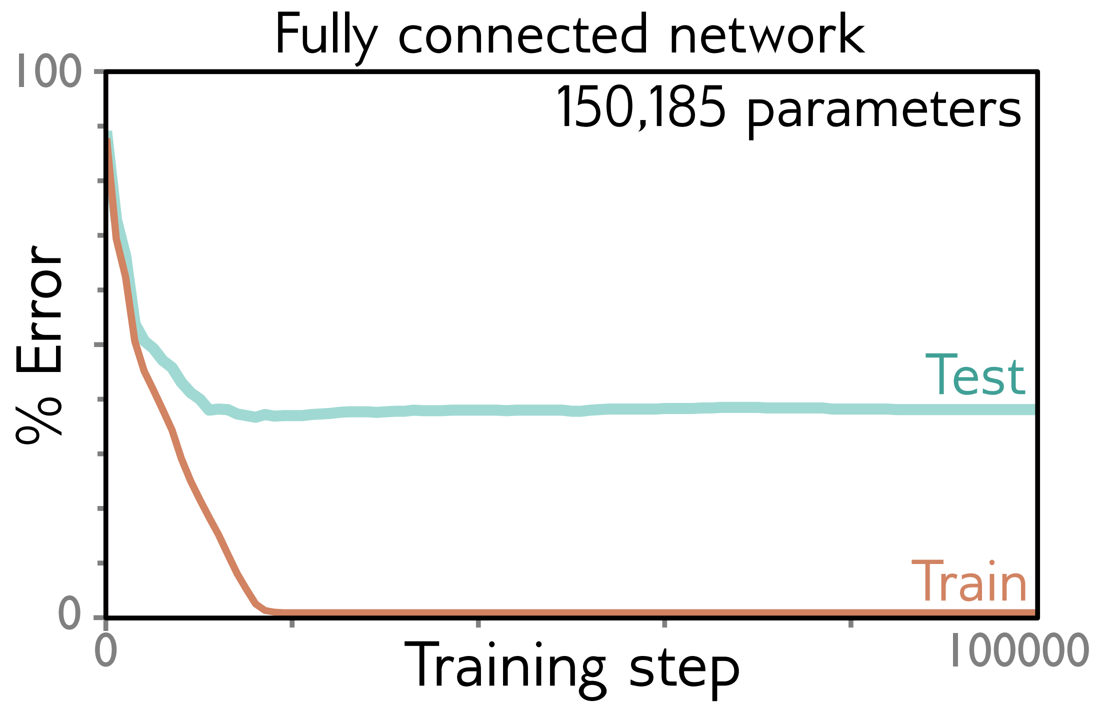


Performance

a)



b)



Why?

- Better inductive bias
- Forced the network to process each location similarly
- Shares information across locations
- Search through a smaller family of input/ouput mappings, all of which are plausible

2D Convolution

Convolution #2

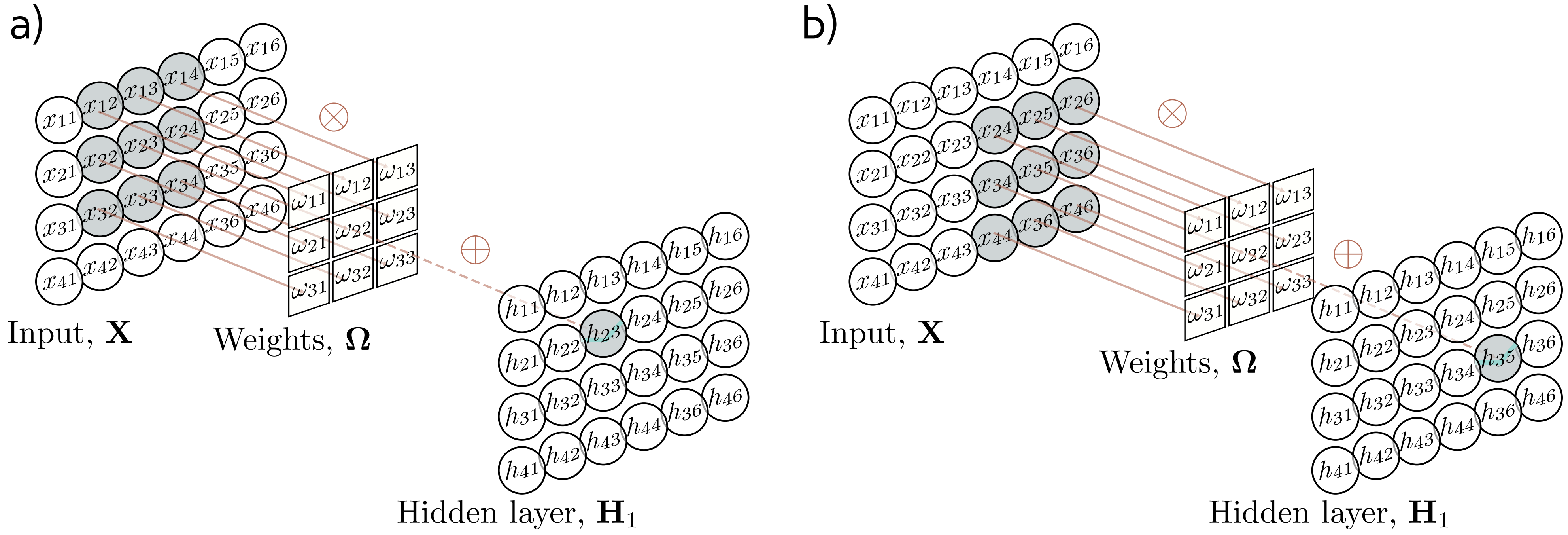
- 2D Convolution
- Downsampling and upsampling, 1x1 convolution
- Image classification
- Object detection
- Semantic segmentation
- Residual networks
- U-Nets and hourglass networks

2D Convolution

- Convolution in 2D
  - Weighted sum over a K x K region
  - K x K weights
- Build into a convolutional layer by adding bias and passing through activation function

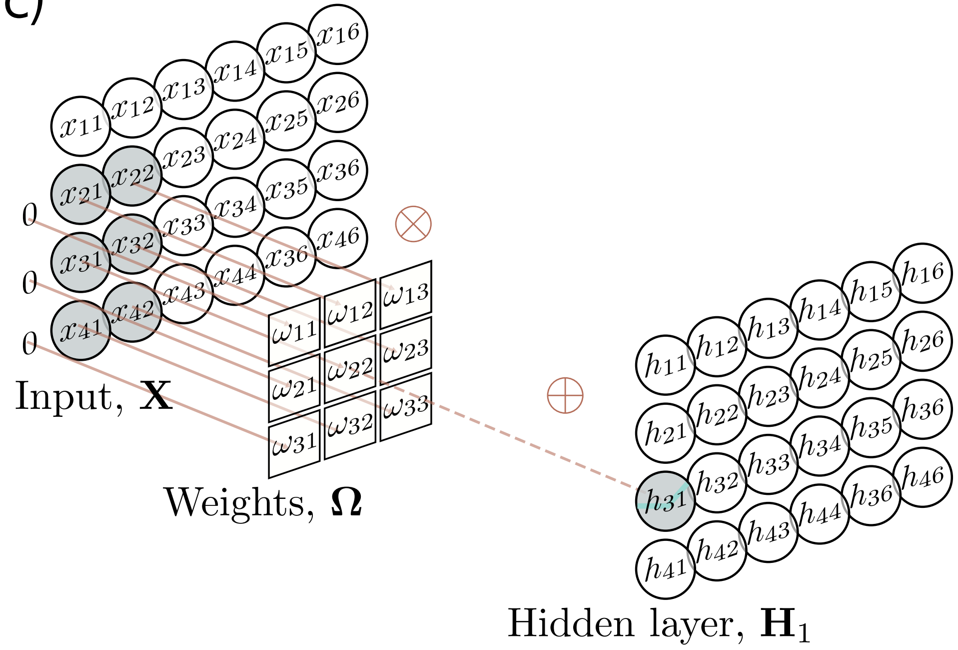
$$h_{i,j} = \mathrm{a}\left[\beta + \sum_{m=1}^{3}\sum_{n=1}^{3}\omega_{m,n}x_{i+m-2,j+n-2}\right]$$

2D Convolution

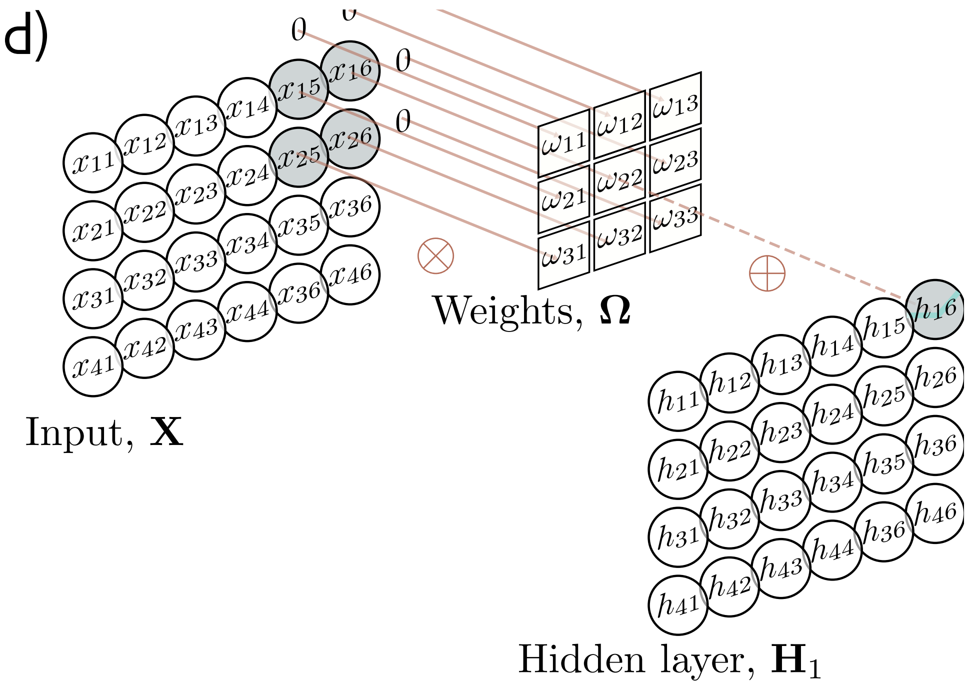


2D Convolution with Zero Padding

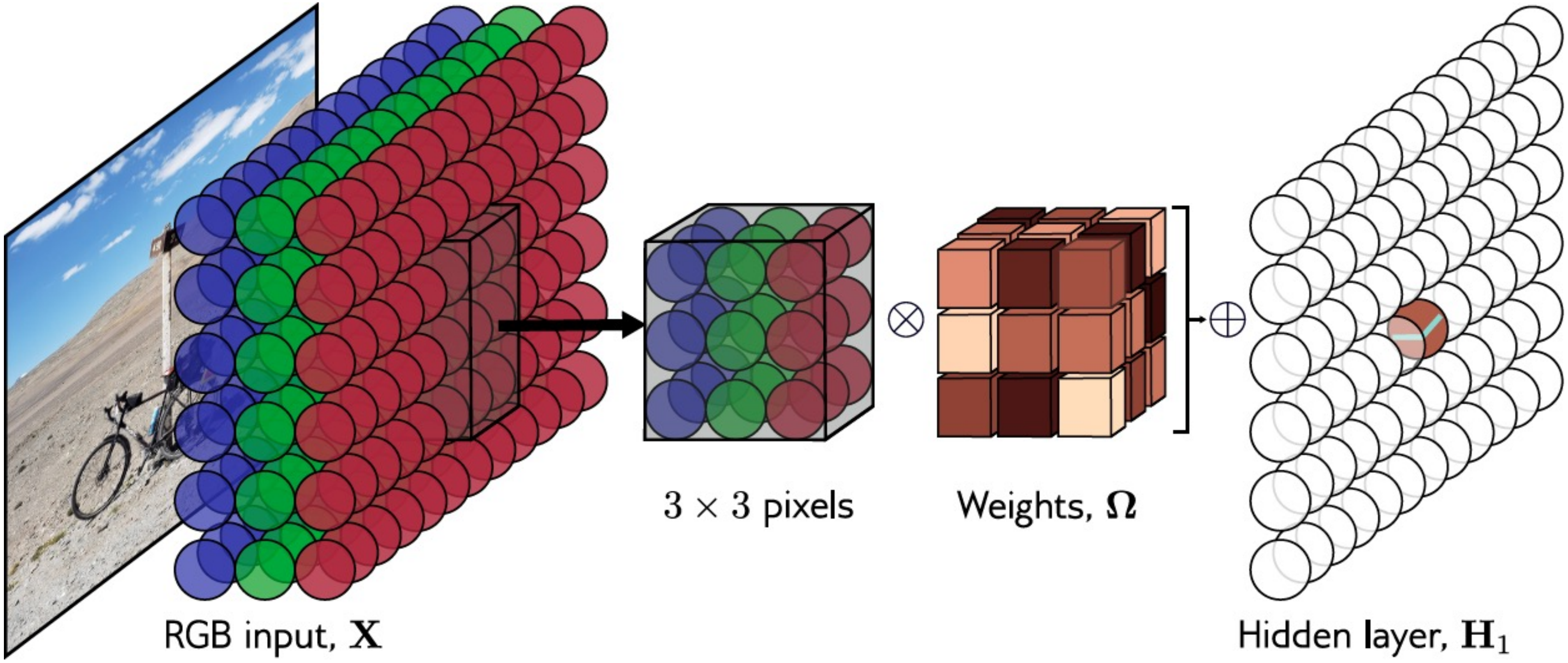
c)



d)



Channels in 2D convolution



Kernel size, stride, dilation all work as you would expect

How many parameters?

- If there are C_i input channels and kernel size K x K

$$\boldsymbol{\omega} \in \mathbb{R}^{C_i \times K \times K} \qquad \boldsymbol{\beta} \in \mathbb{R}$$

- If there are C_i input channels and C_o output channels

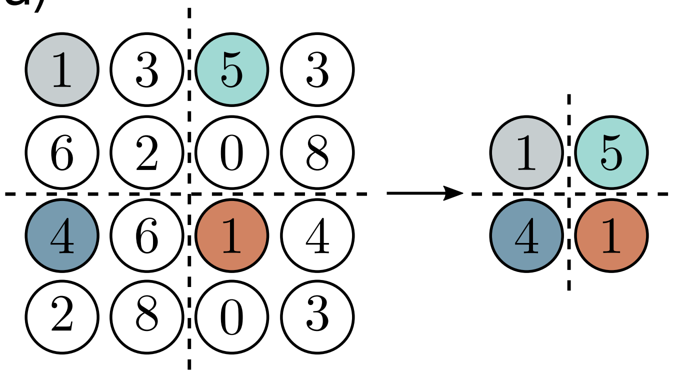
$$\boldsymbol{\omega} \in \mathbb{R}^{C_i \times C_o \times K \times K} \qquad \boldsymbol{\beta} \in \mathbb{R}^{C_o}$$

Convolution #2

- 2D Convolution
- Downsampling and upsampling, 1x1 convolution
- Image classification
- Object detection
- Semantic segmentation
- Residual networks
- U-Nets and hourglass networks

Downsampling

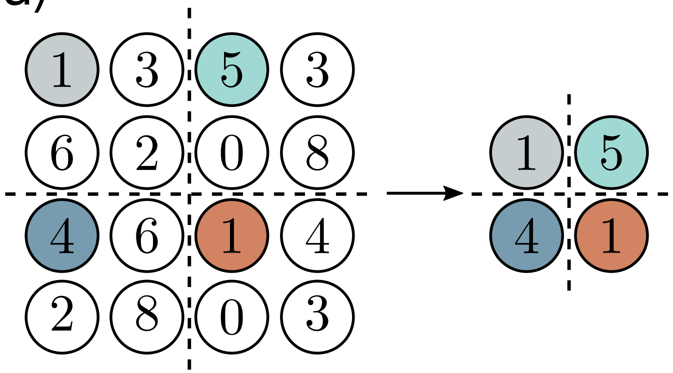
a)



Sample every other position (equivalent to stride two)

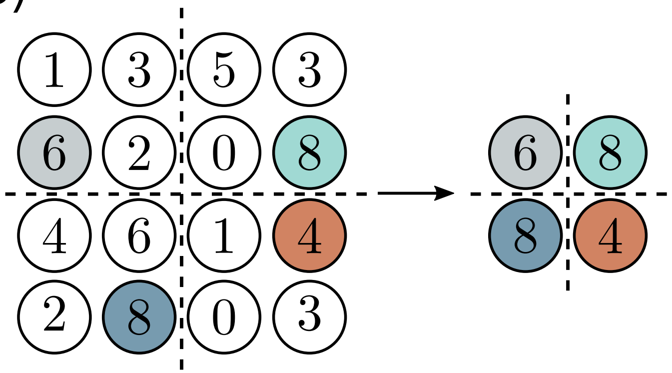
Downsampling

a)



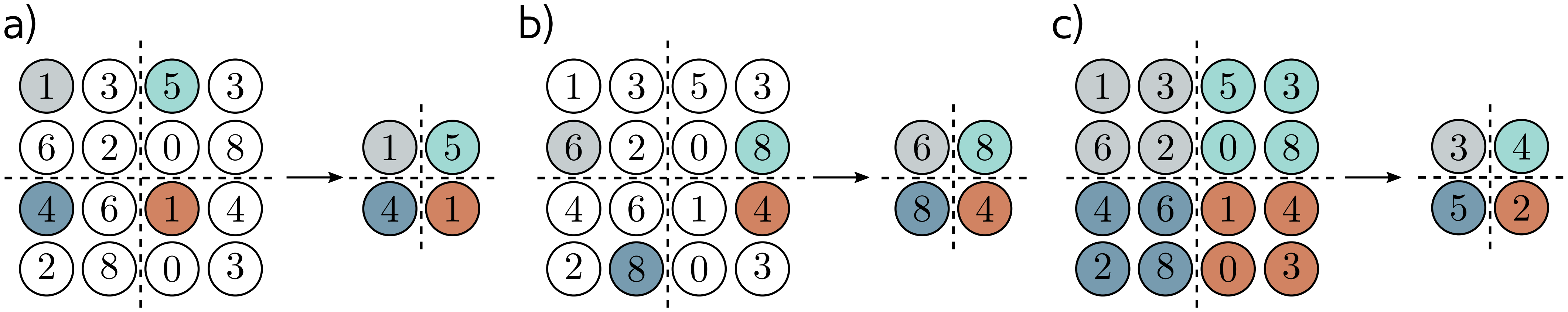
Sample every other position (equivalent to stride two)

b)



Max pooling (partial invariance to translation)

Downsampling



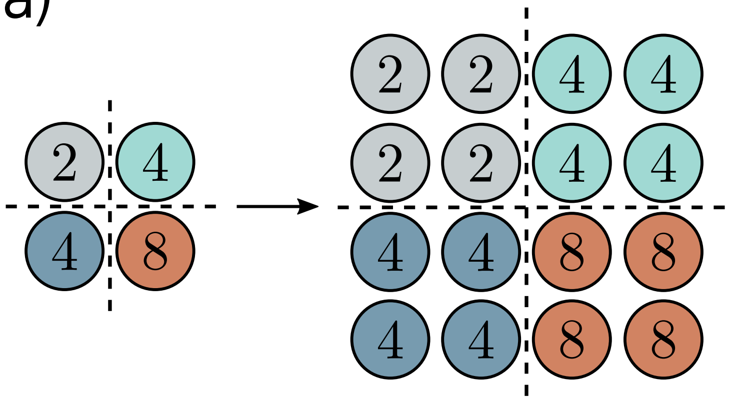
Sample every other position (equivalent to stride two)

Max pooling (partial invariance to translation)

Mean pooling

Upsampling

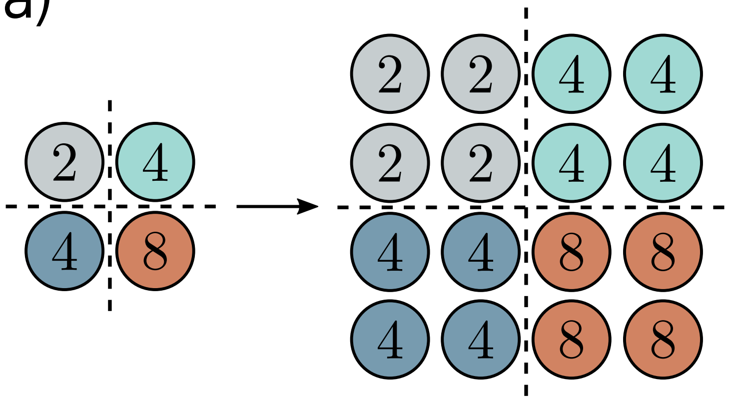
a)



Duplicate

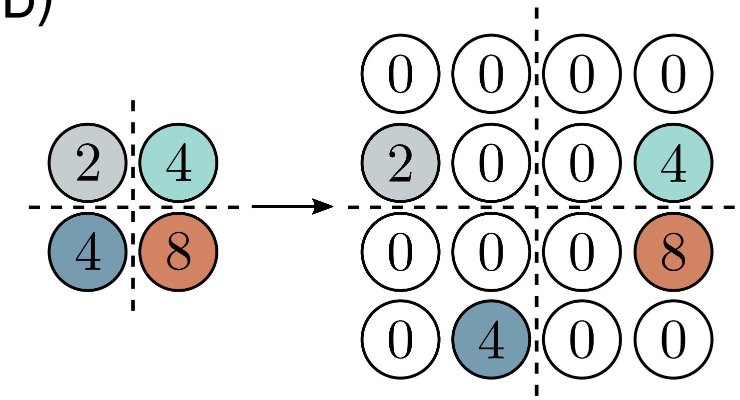
Upsampling

a)



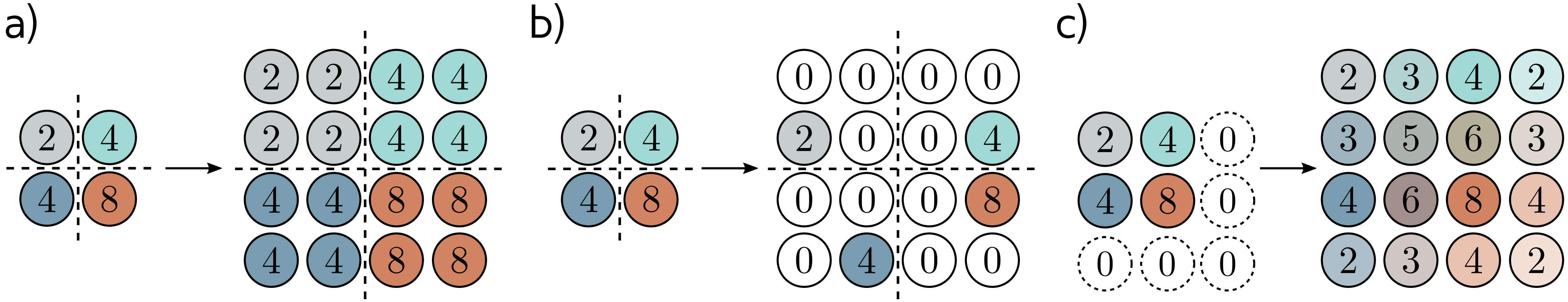
Duplicate

b)



Max-upsampling

Upsampling

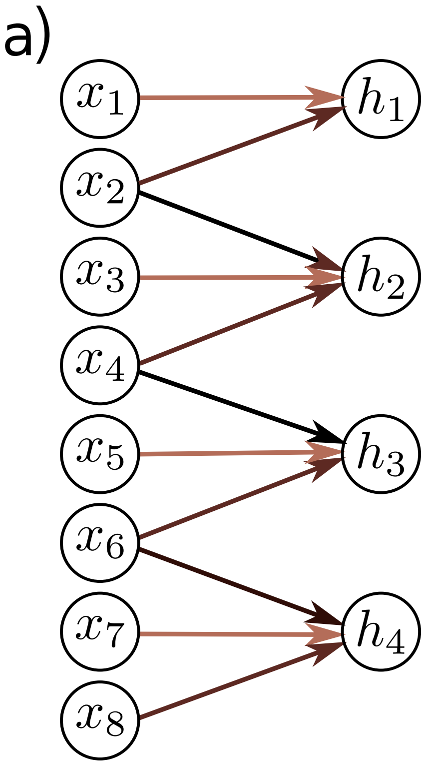


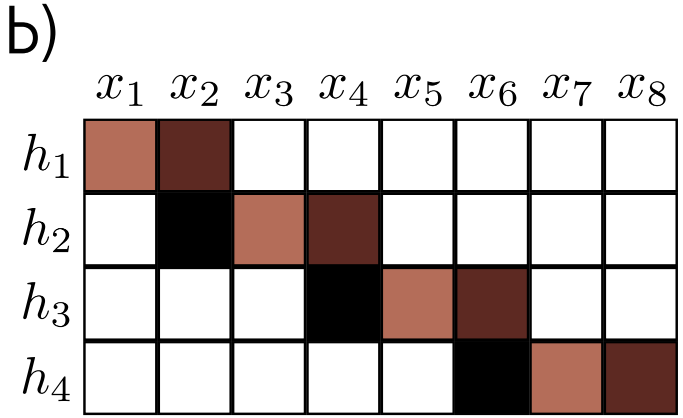
Duplicate

Max-upsampling

Bilinear interpolation

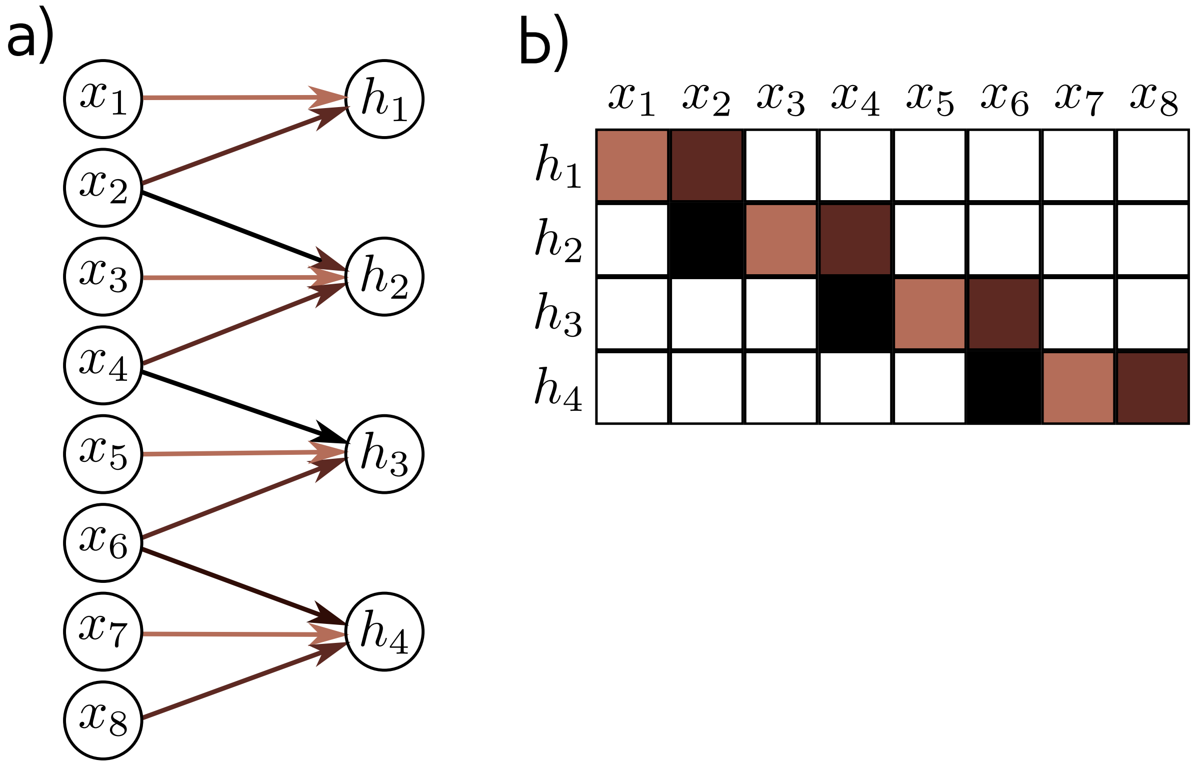
Transposed convolutions



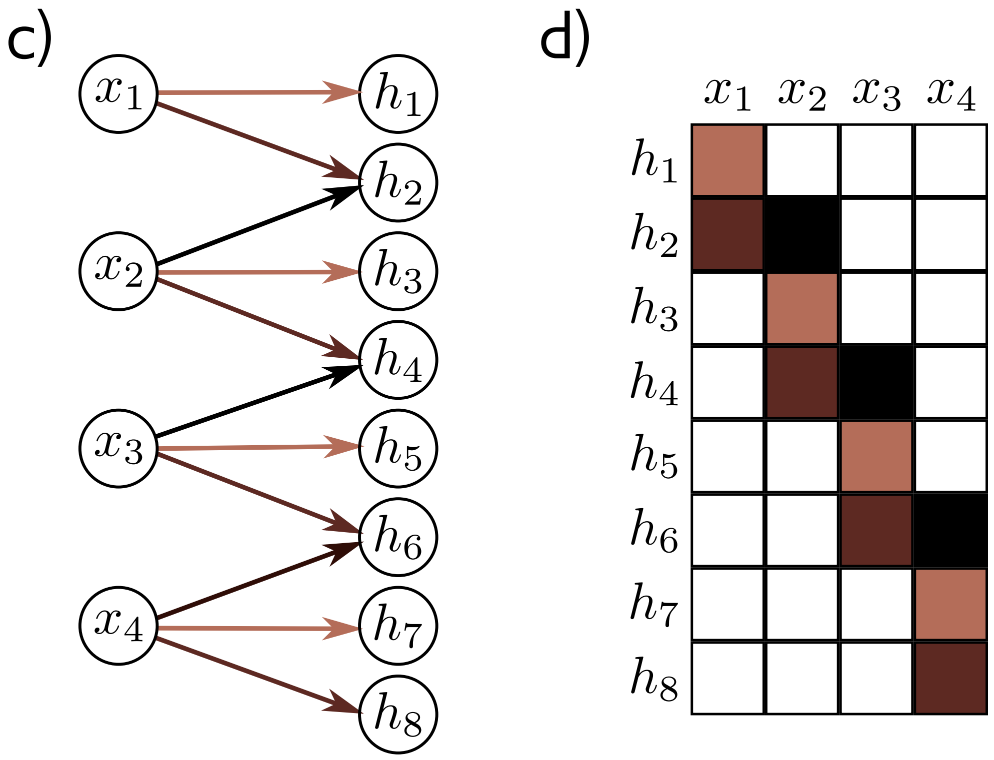


Kernel size 3, Stride 2 convolution

Transposed convolutions

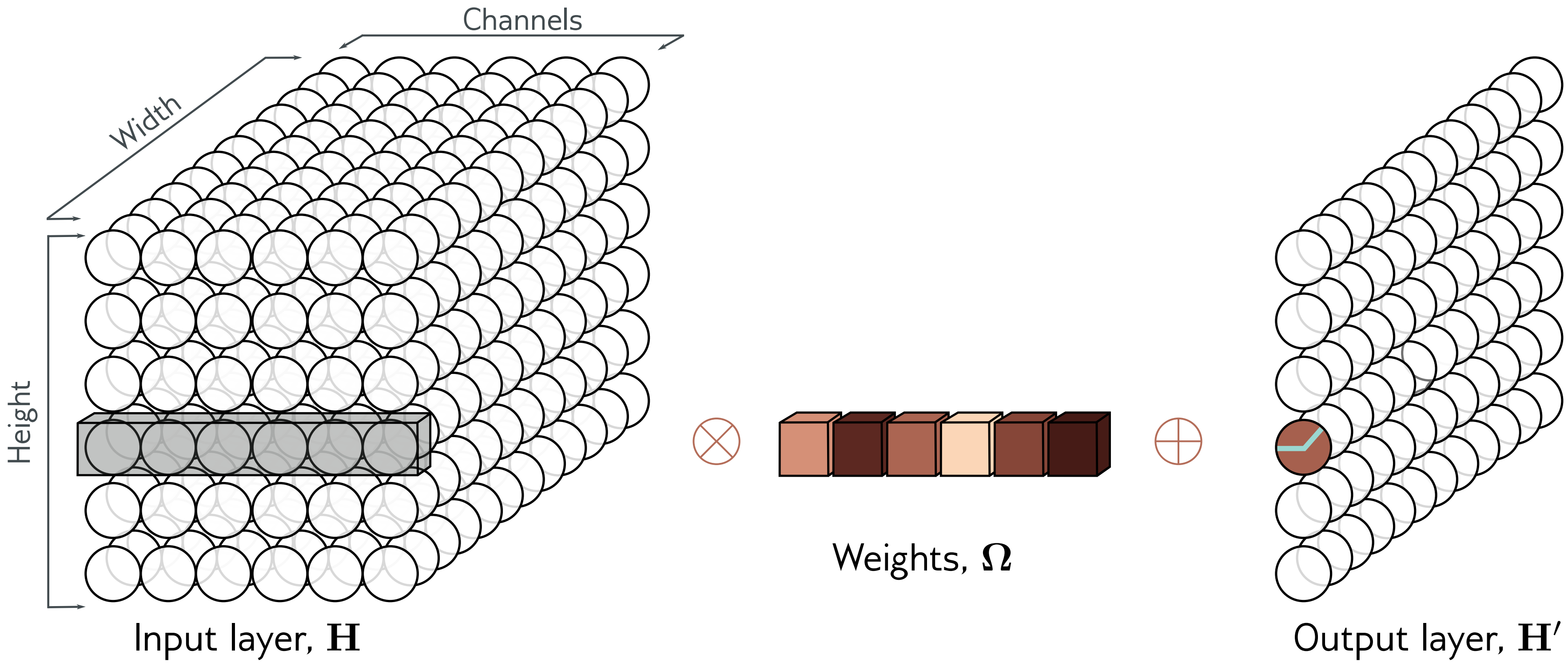


Kernel size 3, Stride 2 convolution



Transposed convolution

1x1 convolution

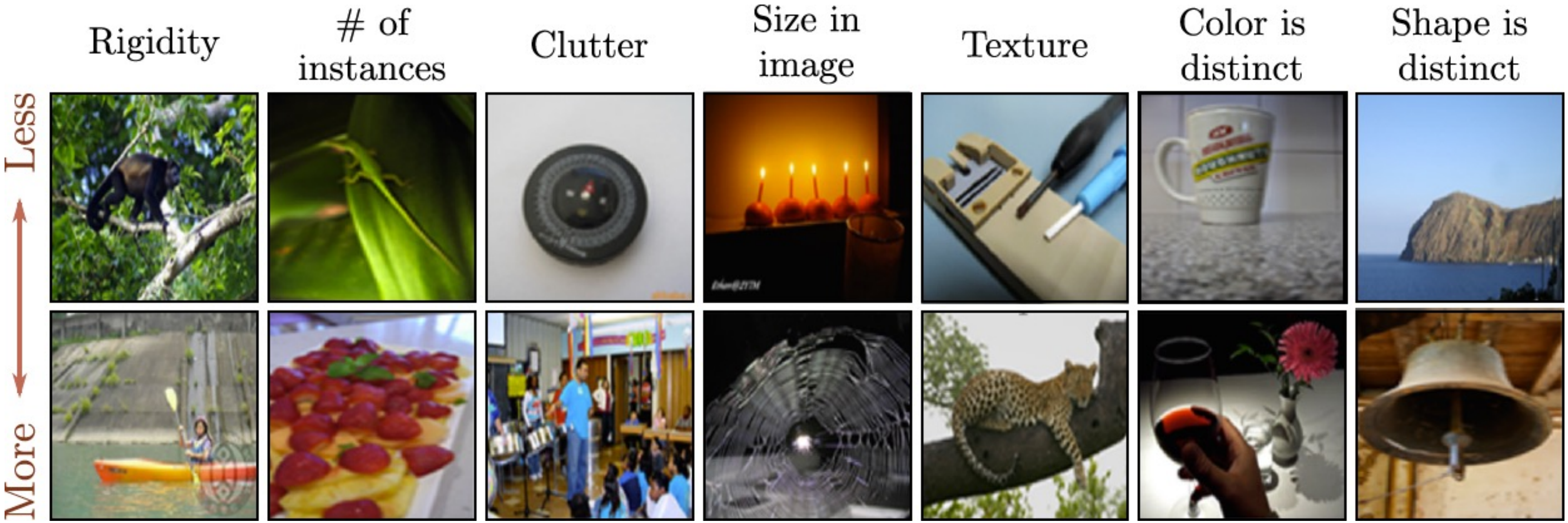


- Mixes channels
- Can change number of channels
- Equivalent to running same fully connected network at each position

Convolution #2

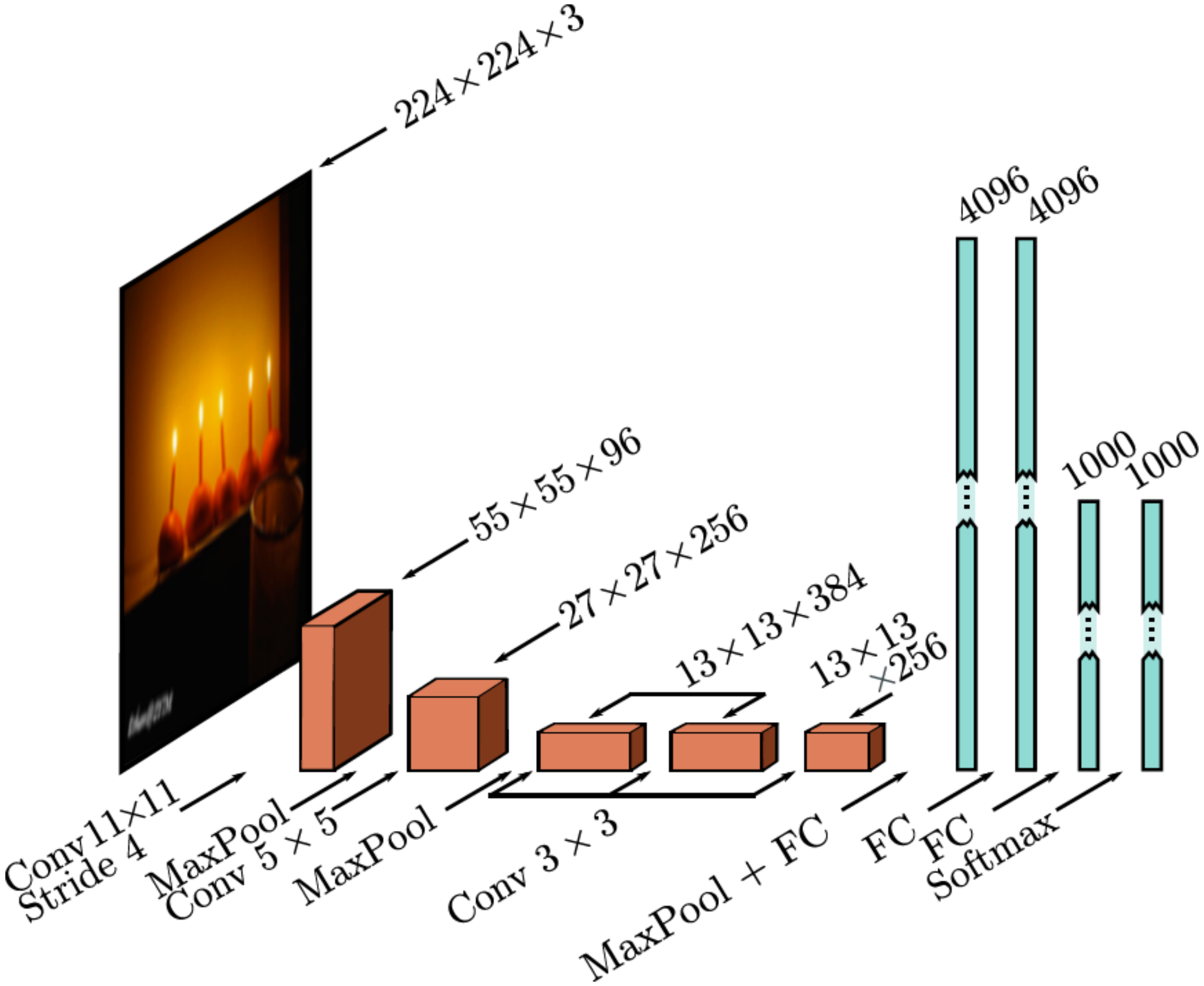
- 2D Convolution
- Downsampling and upsampling, 1x1 convolution
- Image classification
- Object detection
- Semantic segmentation
- Residual networks
- U-Nets and hourglass networks

ImageNet database



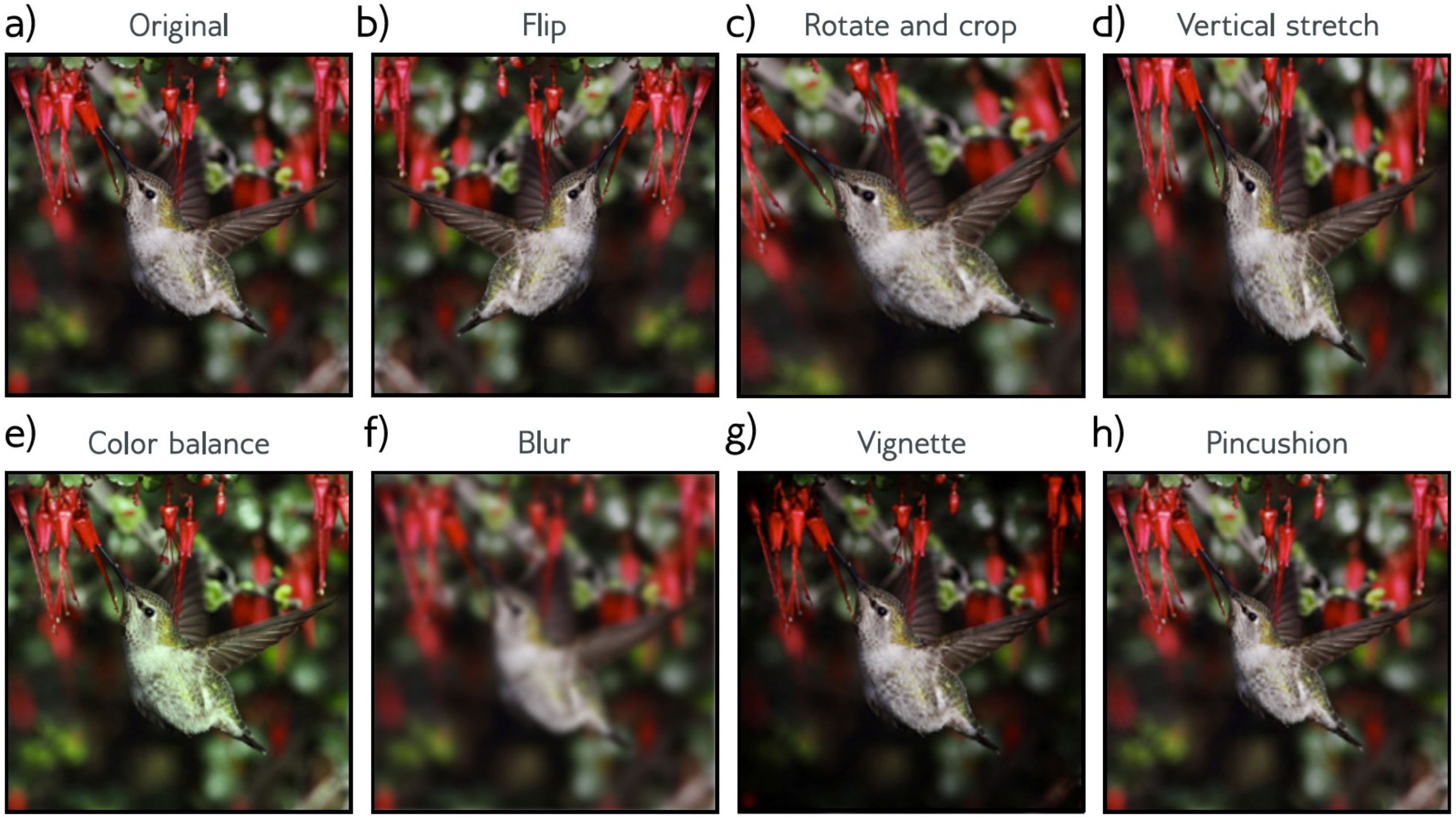
- 224 x 224 images
- 1,281,167 training images, 50,000 validation images, and 100,000 test images
- 1000 classes

AlexNet (2012)



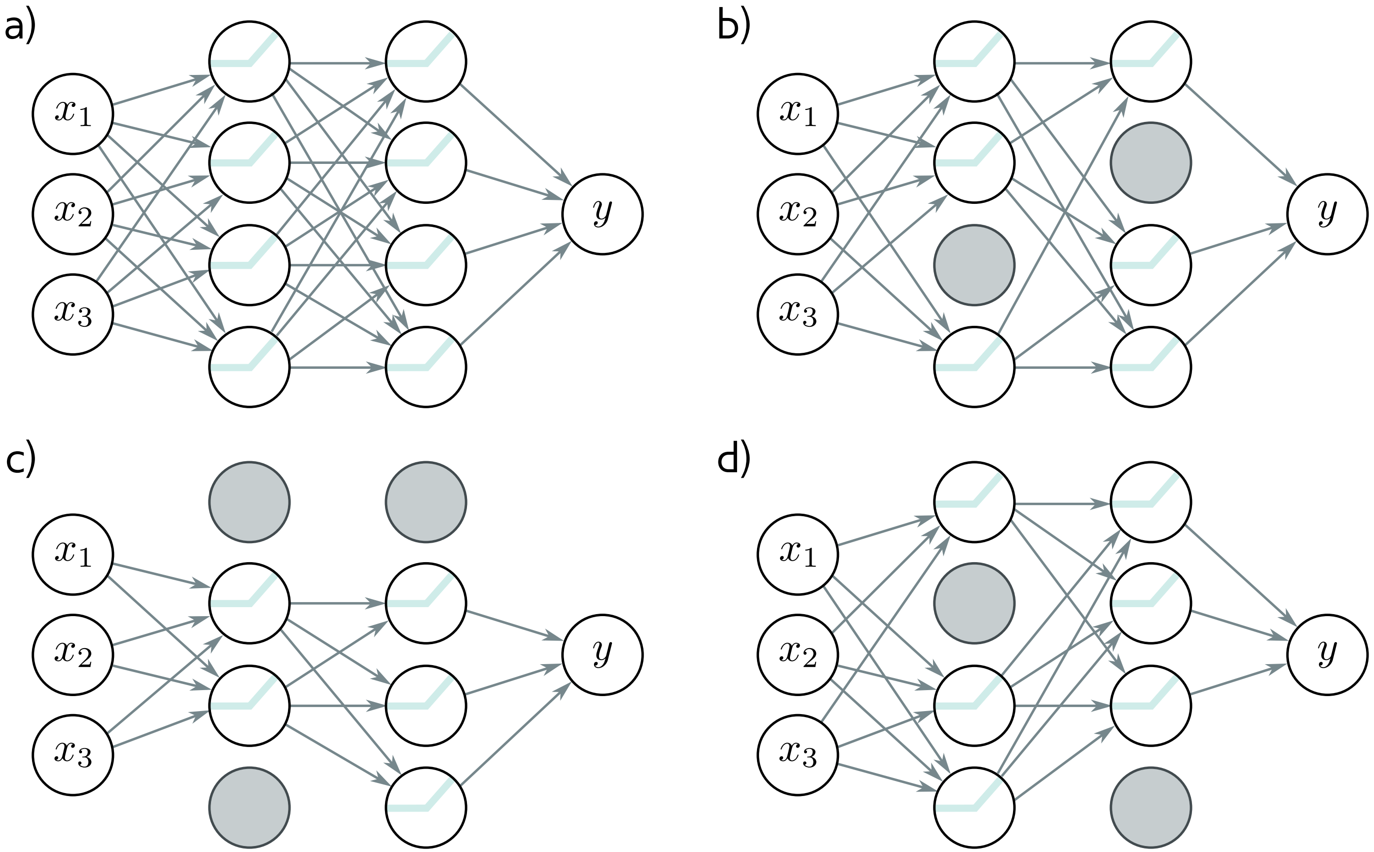
Almost all the 60 million parameters
 parameters are in fully connected layers

Data augmentation



- Data augmentation a factor of 2048 using (i) spatial transformations and (ii) modifications of the input intensities.

Dropout

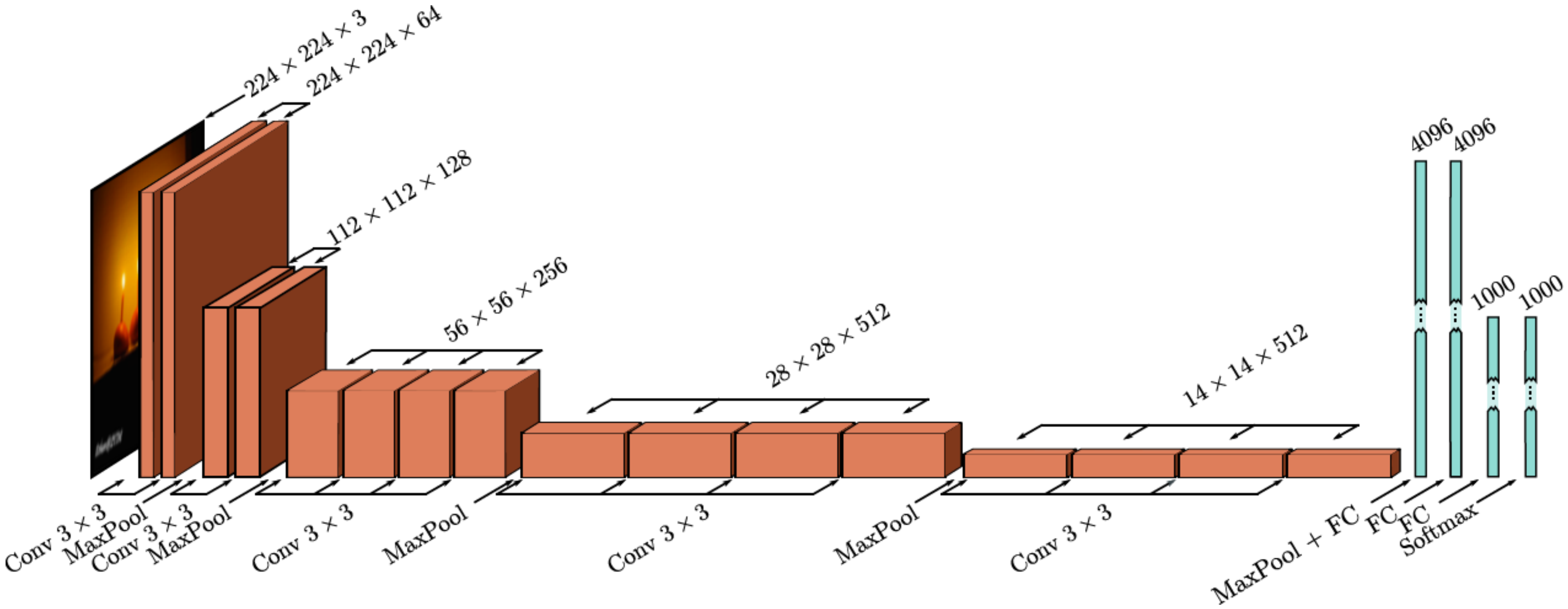


- Dropout was applied in the fully connected layers

Details

- At test time average results from five different cropped and mirrored versions of the image
- SGD with a momentum coefficient of 0.9 and batch size of 128.
- L2 (weight decay) regularizer used.
- This system achieved a 16.4% top-5 error rate and a 38.1% top-1 error rate.

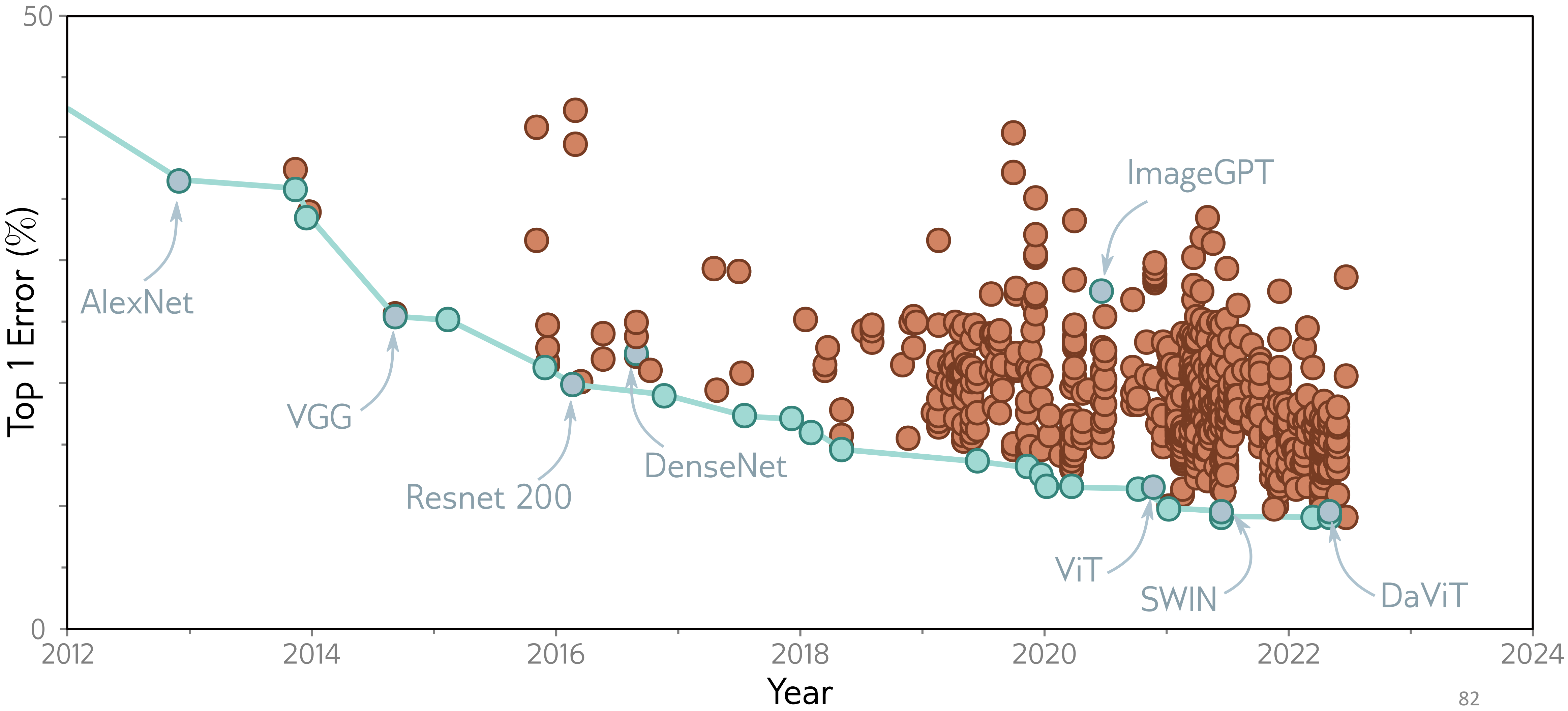
VGG (2015)


$224 \times 224 \times 3$
$224 \times 224 \times 64$
$112 \times 112 \times 128$
$56 \times 56 \times 256$
$28 \times 28 \times 512$
$14 \times 14 \times 512$
4096
4096
1000
1000
Conv 3×3
MaxPool
Conv 3×3
MaxPool
Conv 3×3
MaxPool
Conv 3×3
MaxPool
Conv 3×3
MaxPool + FC
FC
FC
Softmax

Details

- 19 hidden layers
- 144 million parameters
- 6.8% top-5 error rate, 23.7% top-1 error rate

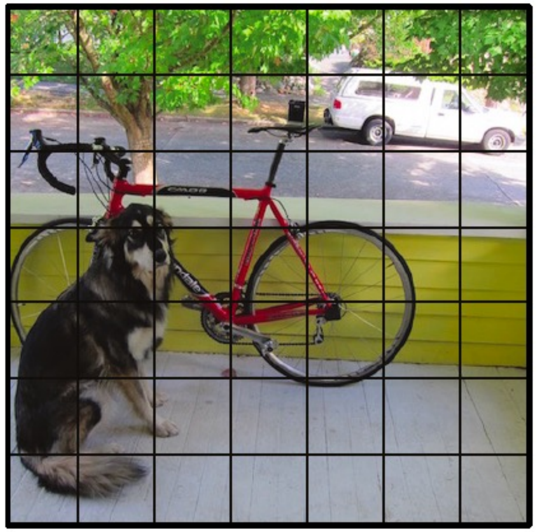
ImageNet History



Convolution #2

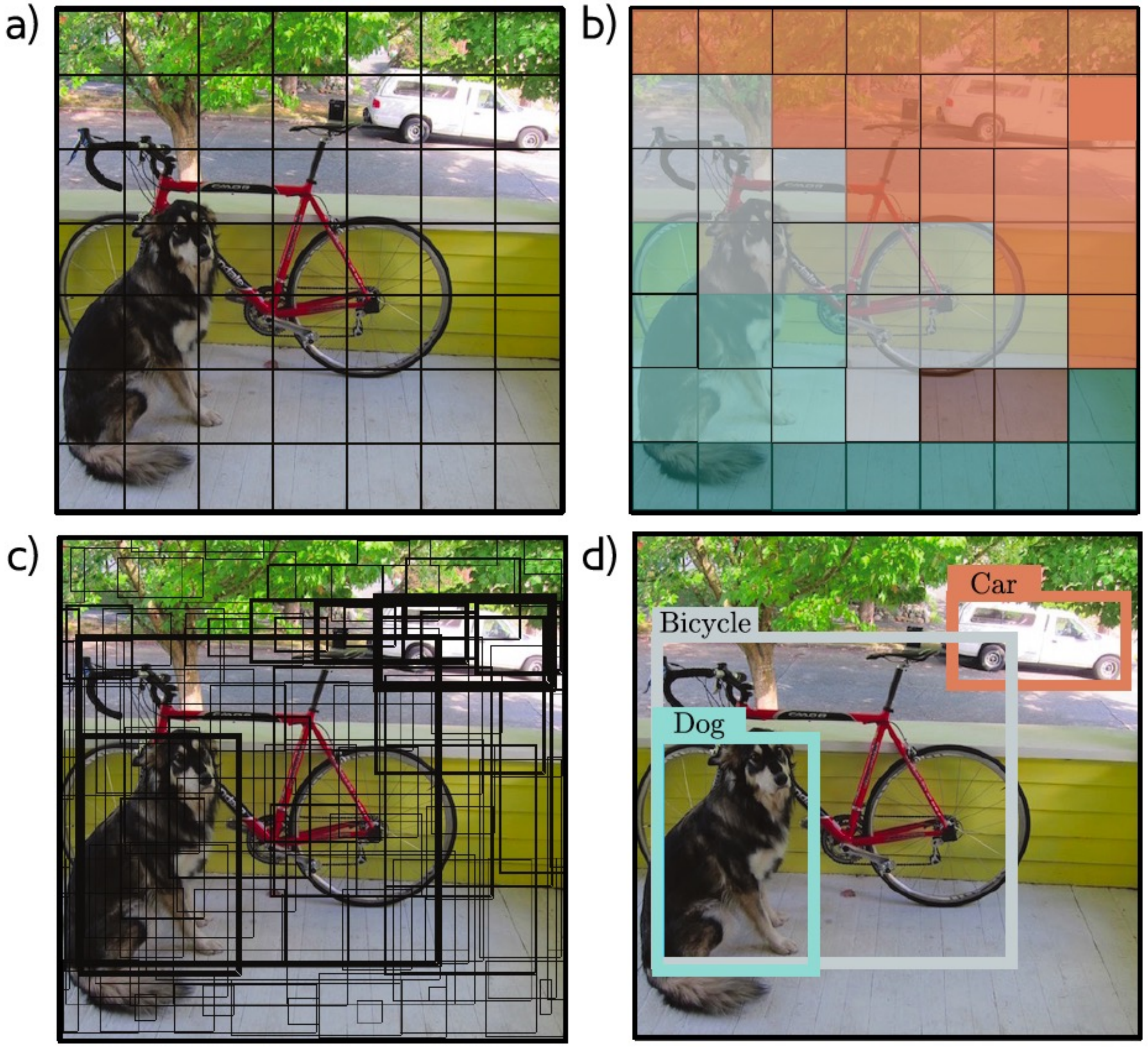
- 2D Convolution
- Downsampling and upsampling, 1x1 convolution
- Image classification
- Object detection
- Semantic segmentation
- Residual networks
- U-Nets and hourglass networks

You Only Look Once (YOLO)

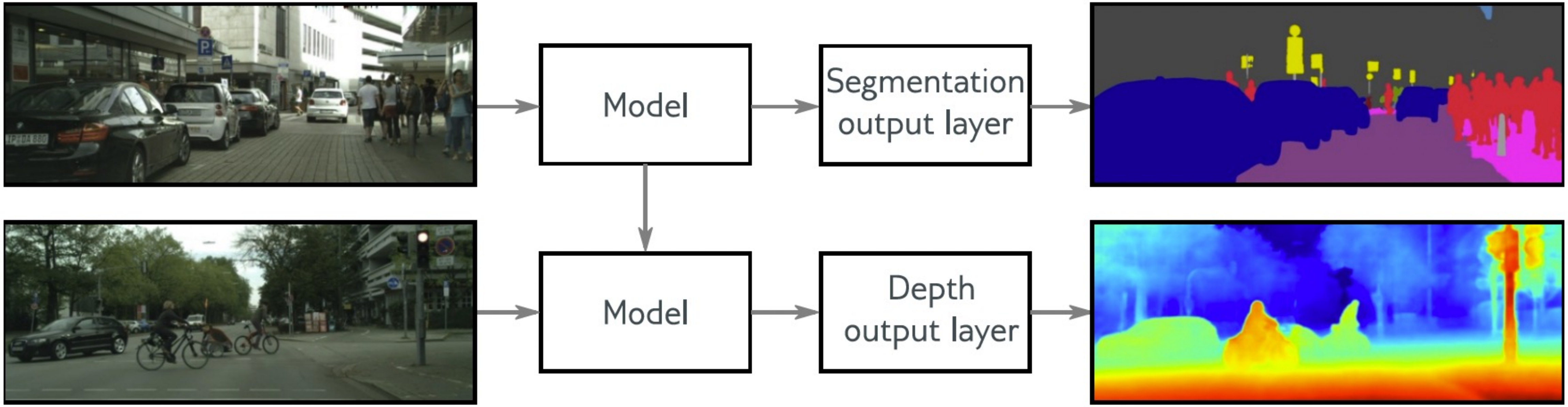


- Network similar to VGG (448x448 input)
- 7×7 grid of locations
- Predict class at each location
- Predict 2 bounding boxes at each location
  - Five parameters –x,y, height, width, and confidence
- Momentum, weight decay, dropout, and data augmentation
- Heuristic at the end to threshold and decide final boxes – (non maximum suppression)

Object detection (YOLO)

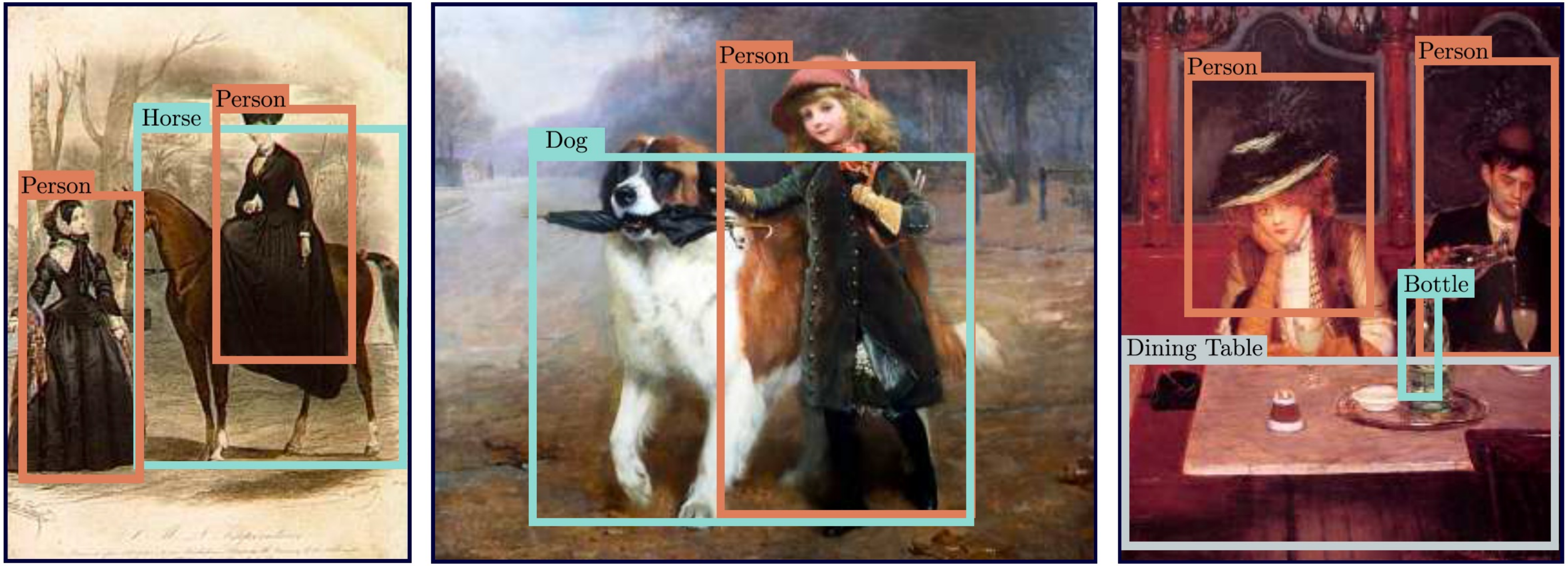


Transfer learning



Transfer learning from ImageNet classification

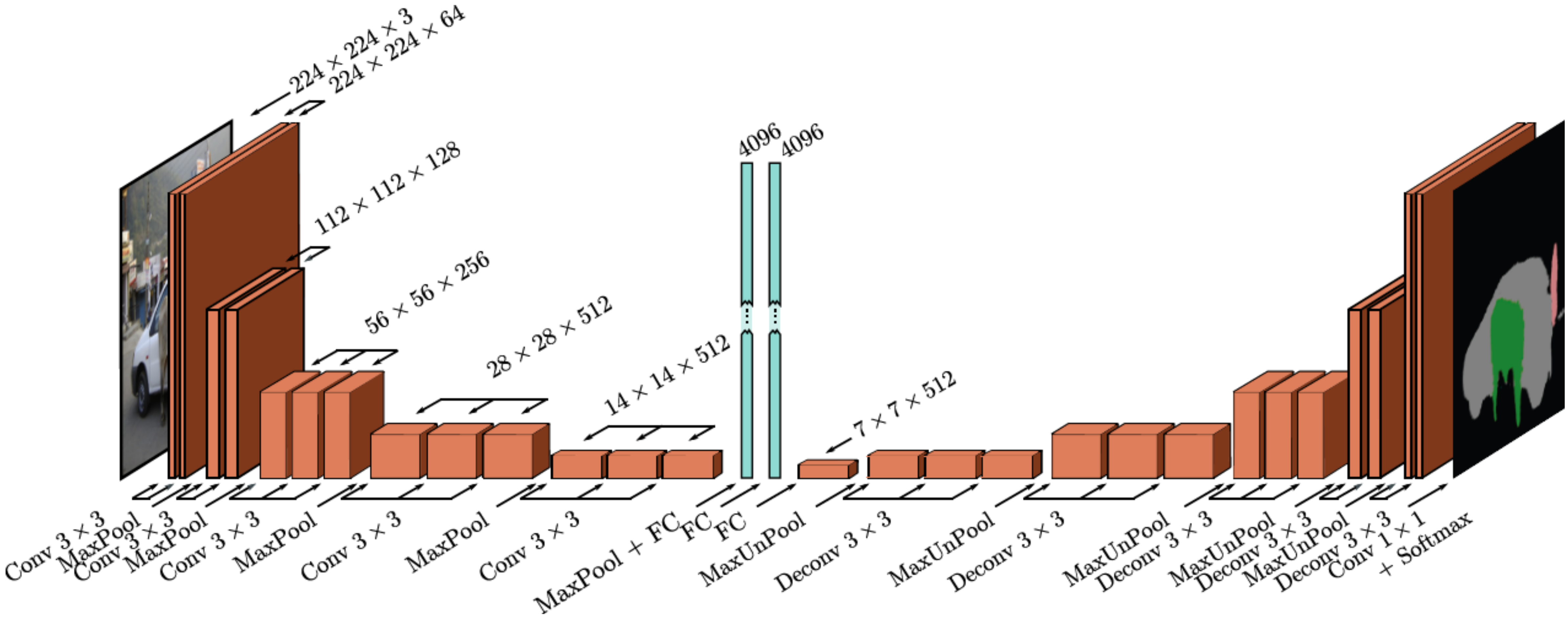
Results


Person
Horse
Person
Person
Dog
Person
Person
Bottle
Dining Table

Convolution #2

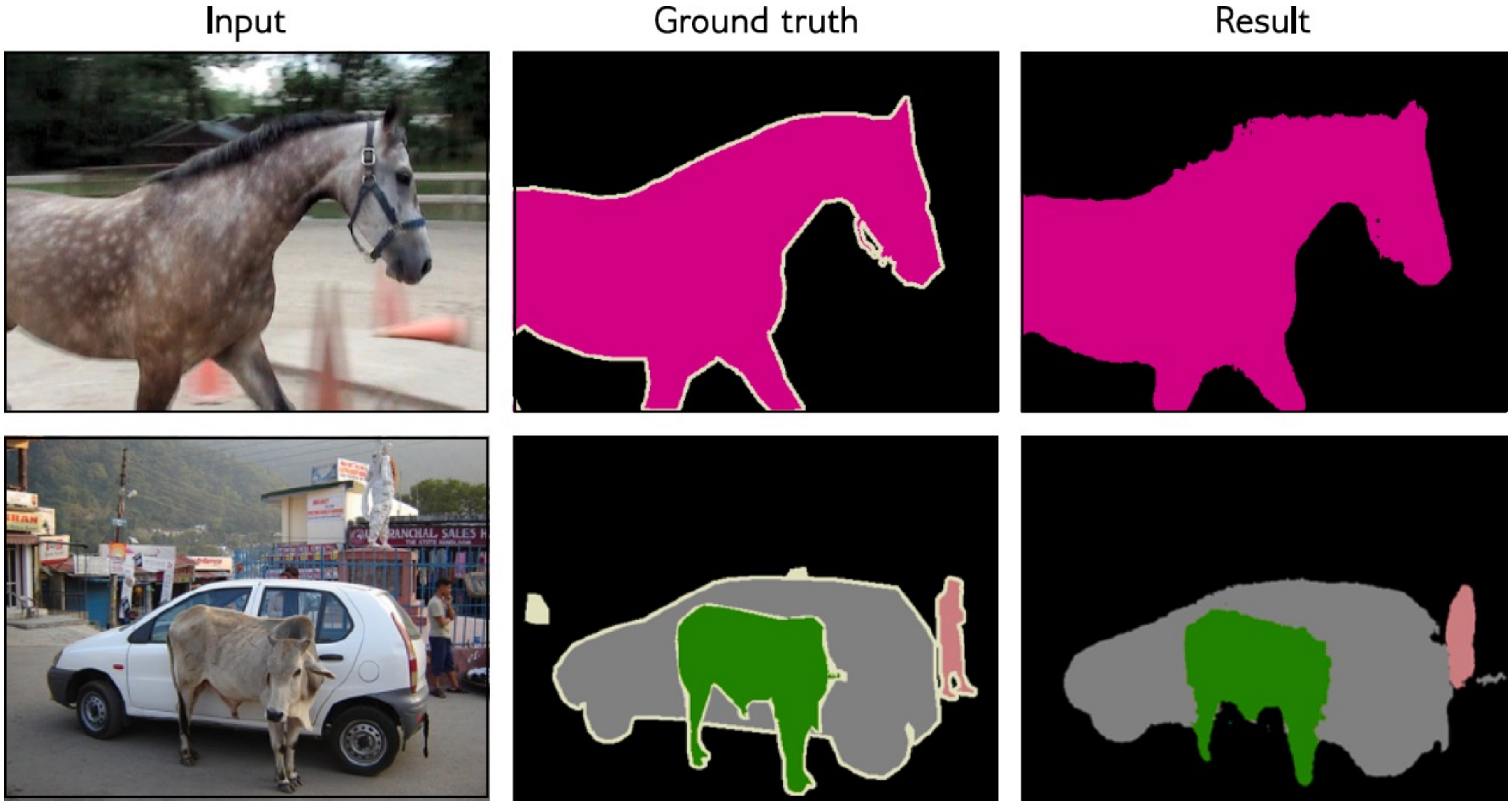
- 2D Convolution
- Downsampling and upsampling, 1x1 convolution
- Image classification
- Object detection
- Semantic segmentation
- Residual networks
- U-Nets and hourglass networks

Semantic Segmentation (2015)



Encoder Decoder

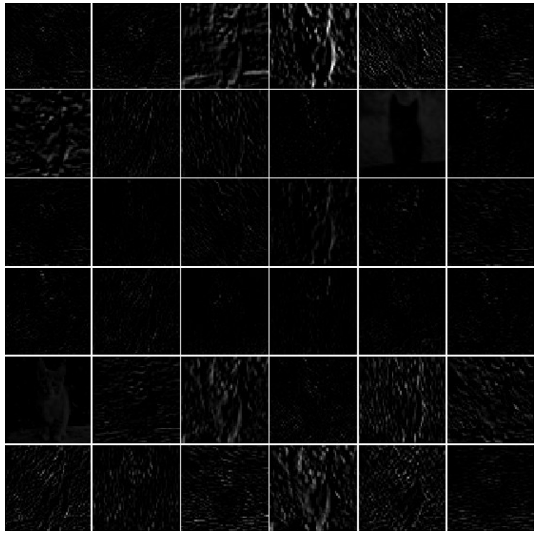
Semantic segmentation results



AlexNet

Cat image input
(not actual image)

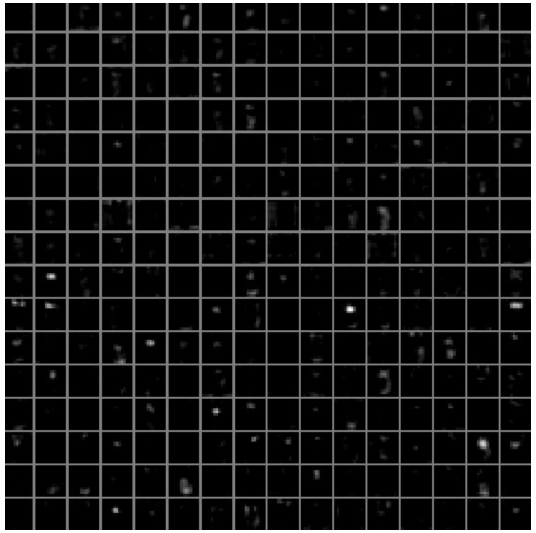
Activations
(feature maps)



1st Layer

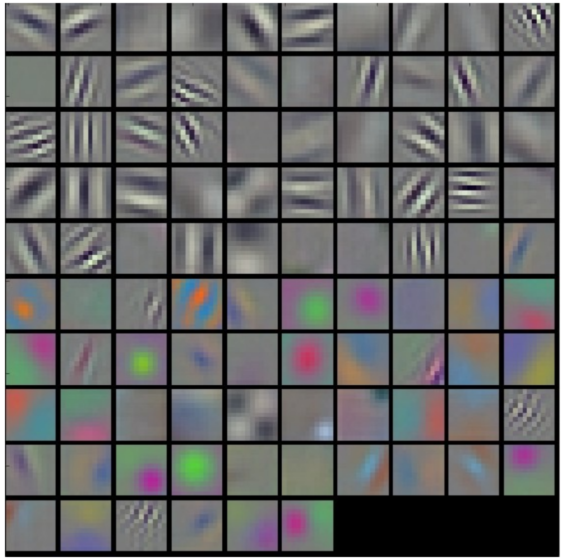
...

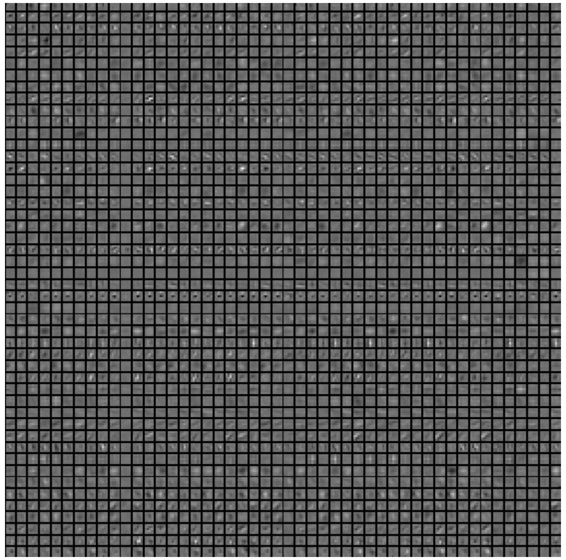
2nd Layer



5th Layer

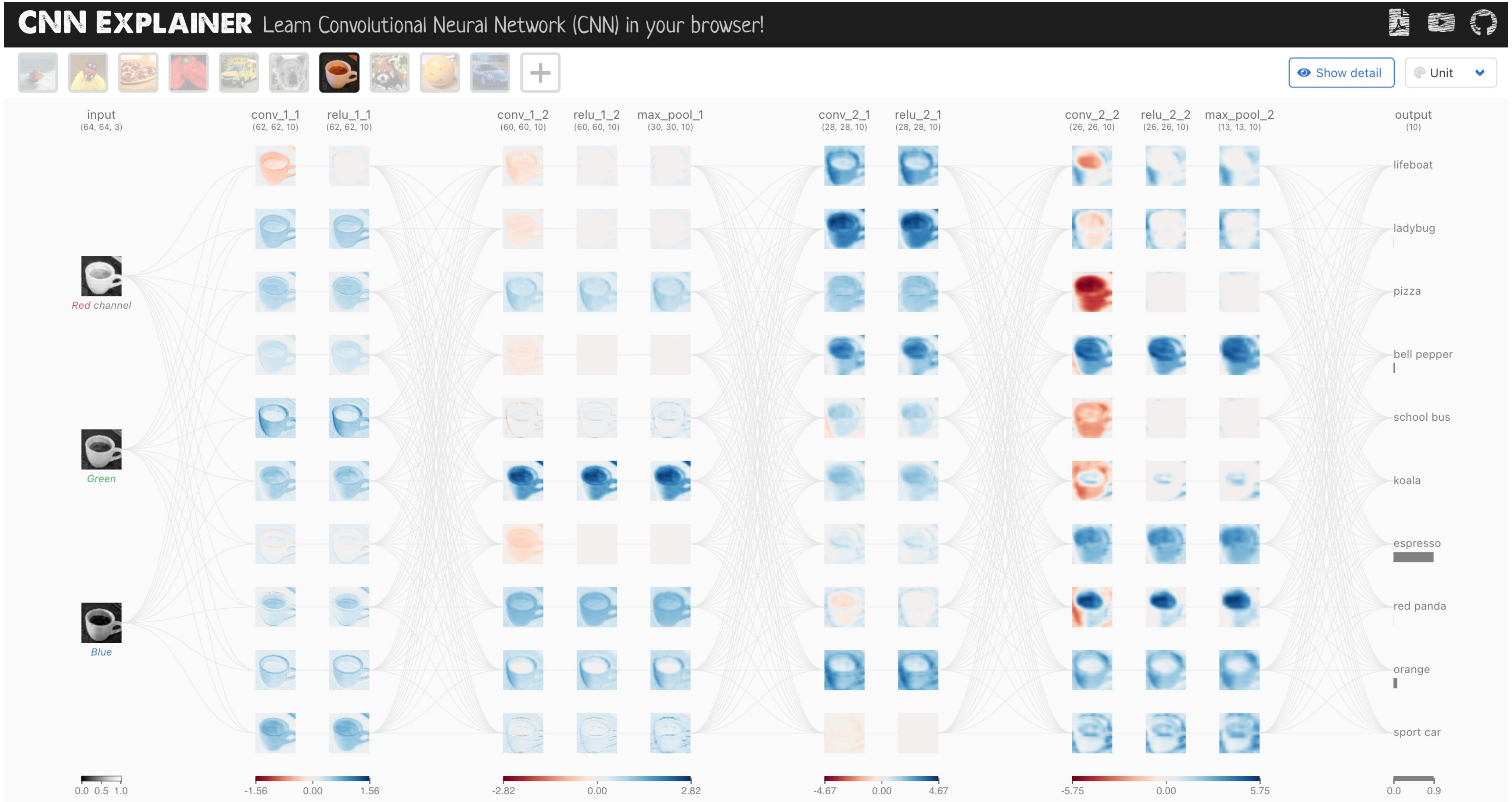
Filter Kernels





...

https://cs231n.github.io/understanding-cnn/



https://poloclub.github.io/cnn-explainer/

Feedback?

[Link]