BOSTON UNIVERSITY

Deep Learning for Data Science
DS 542

https://dl4ds.github.io/fa2025/

Transformer Details

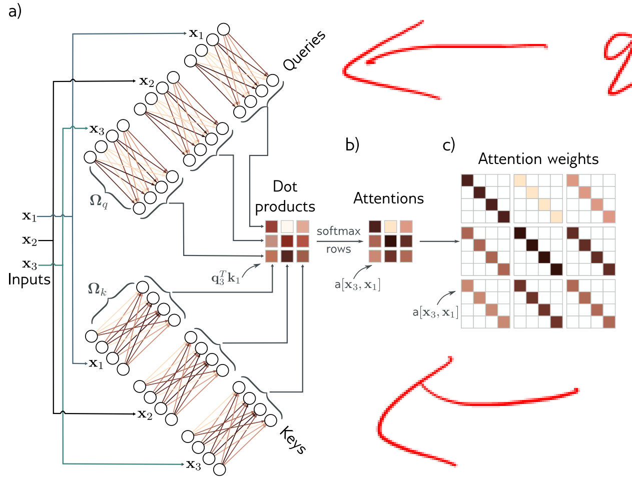
Original slides by Tom Gardos,
other content from Understanding Deep Learning unless otherwise cited

Plan for Today

- Transformer recap
- What are tokens?
- Tokenization and word embedding
- Next token selection
- Transformers for long sequences

Recap From Part 1

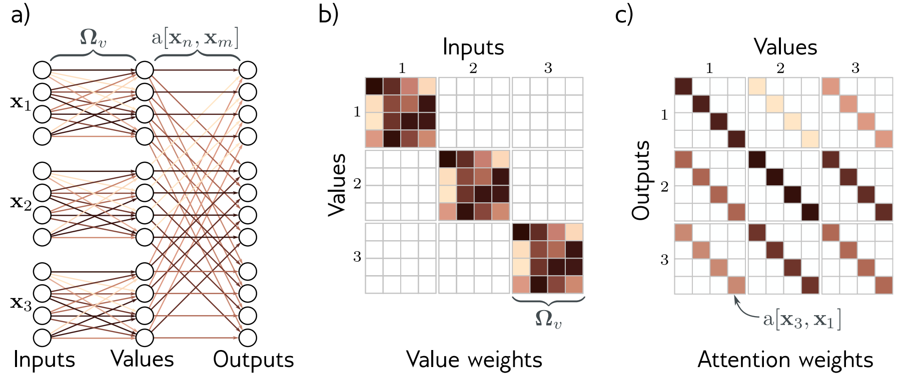
- Motivation
- Dot-product self-attention
- Applying Self-Attention
- The Transformer Architecture
- Three Types of NLP Transformer Models
  - Encoder
  - Decoder
  - Encoder-Decoder



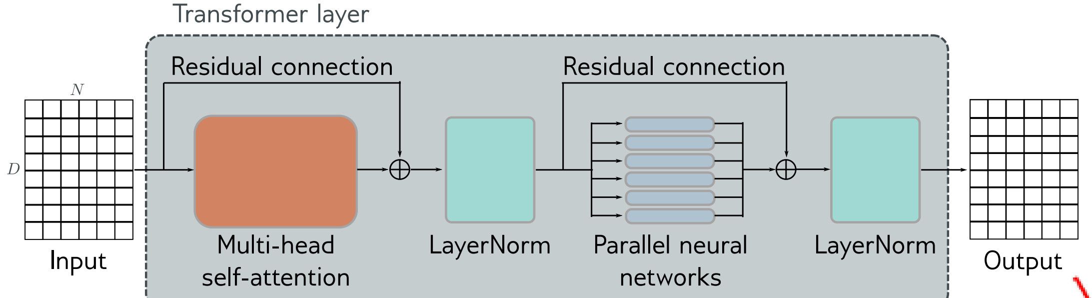
queries

keys

many inputs



smaller than FC



Transformer block repeated a lot

3 Types of Transformer Models

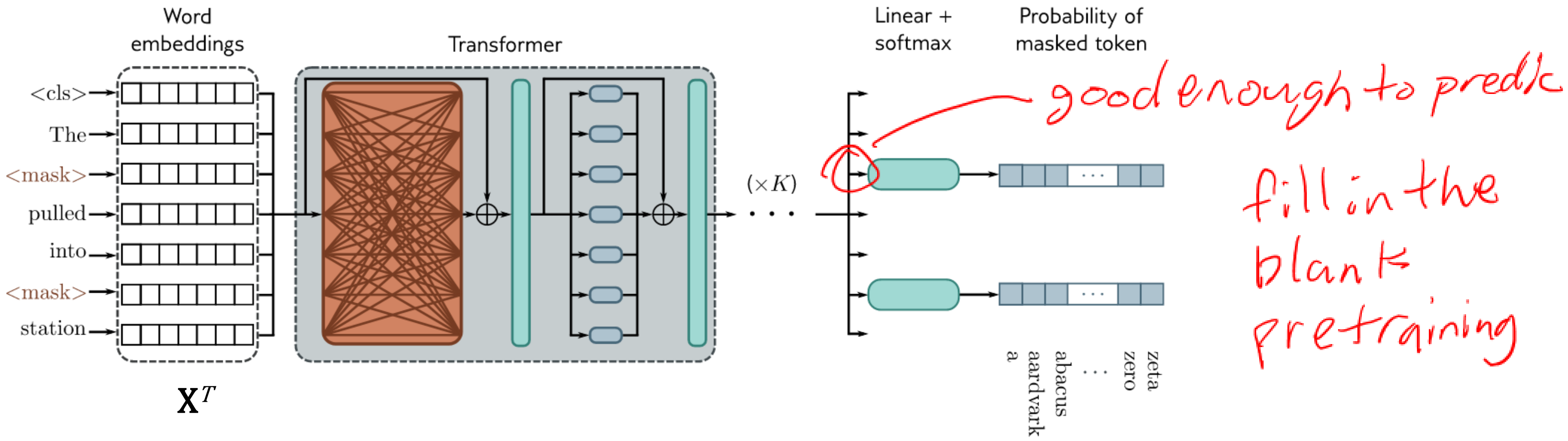
1. *Encoder* – transforms text embeddings into representations that support variety of tasks (e.g. sentiment analysis, classification)
   - Model Example: BERT

2. *Decoder* – predicts the next token to continue the input text (e.g. ChatGPT, AI assistants)
   - Model Example: GPT4, GPT4

3. *Encoder-Decoder* – used in sequence-to-sequence tasks, where one text string is converted to another (e.g. machine translation)

Encoder Pre-Training

Special <cls> token used for aggregate sequence representation for classification

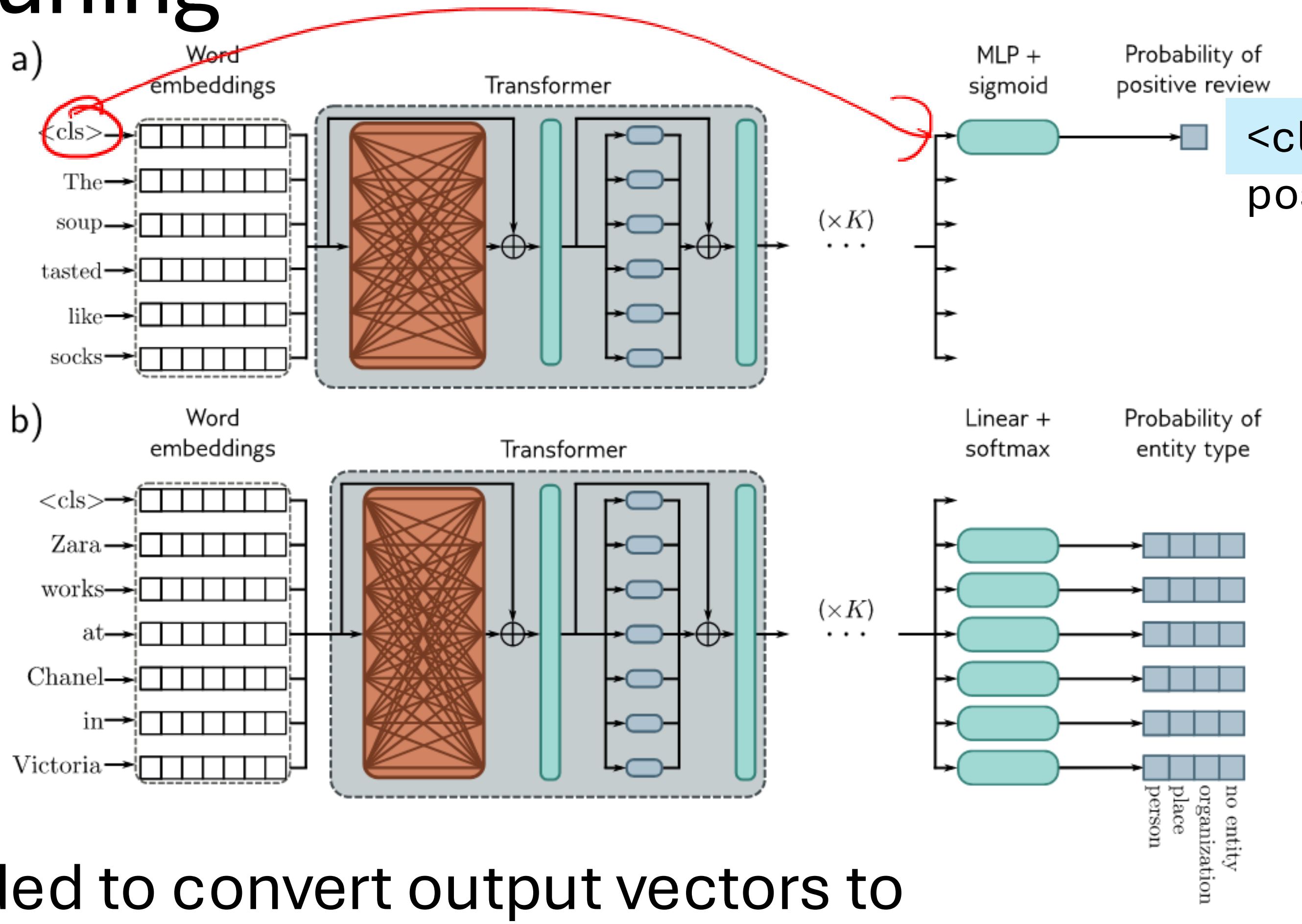


- A small percentage of input embedding replaced with a generic <mask> token
- Predict missing token from output embeddings
- Added linear layer and softmax to generate probabilities over vocabulary
- Trained on BooksCorpus (800M words) and English Wikipedia (2.5B words)

Encoder Fine-Tuning

Sentiment Analysis

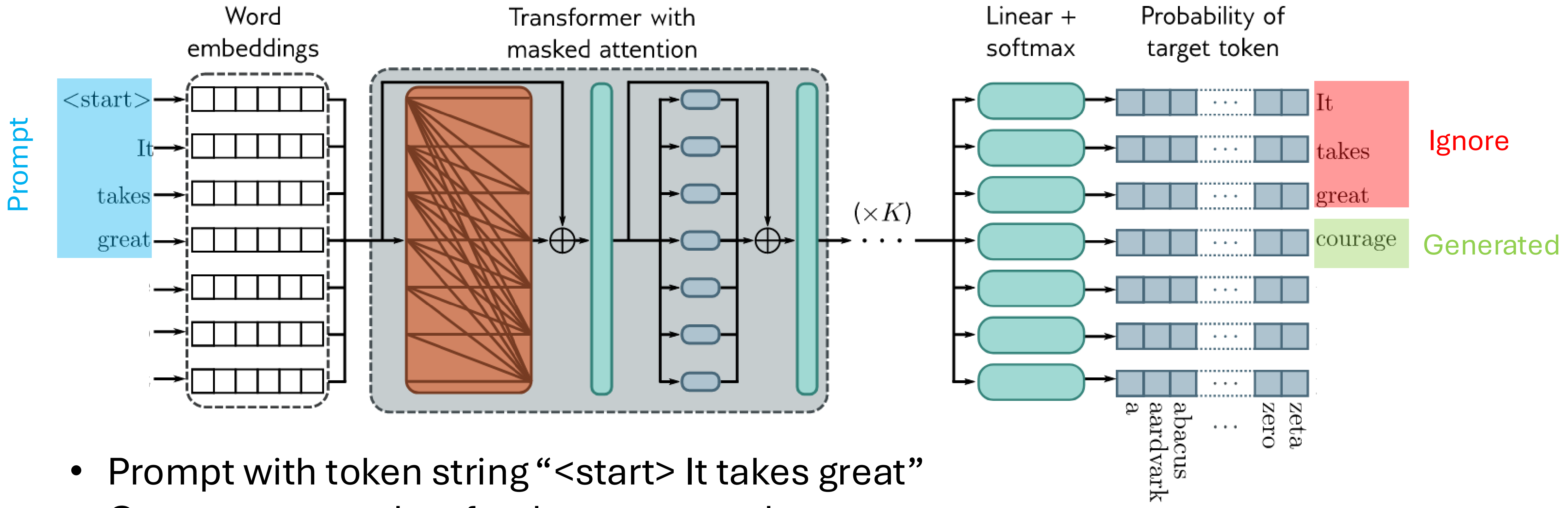
Named Entity Recognition (NER)



<cls> token position

- Extra layer(s) appended to convert output vectors to desired output format
- 3rd Example: Text span prediction -- predict start and end location of answer to a question in passage of Wikipedia, see https://rajpurkar.github.io/SQuAD-explorer/

Decoder: Text Generation (Generative AI)



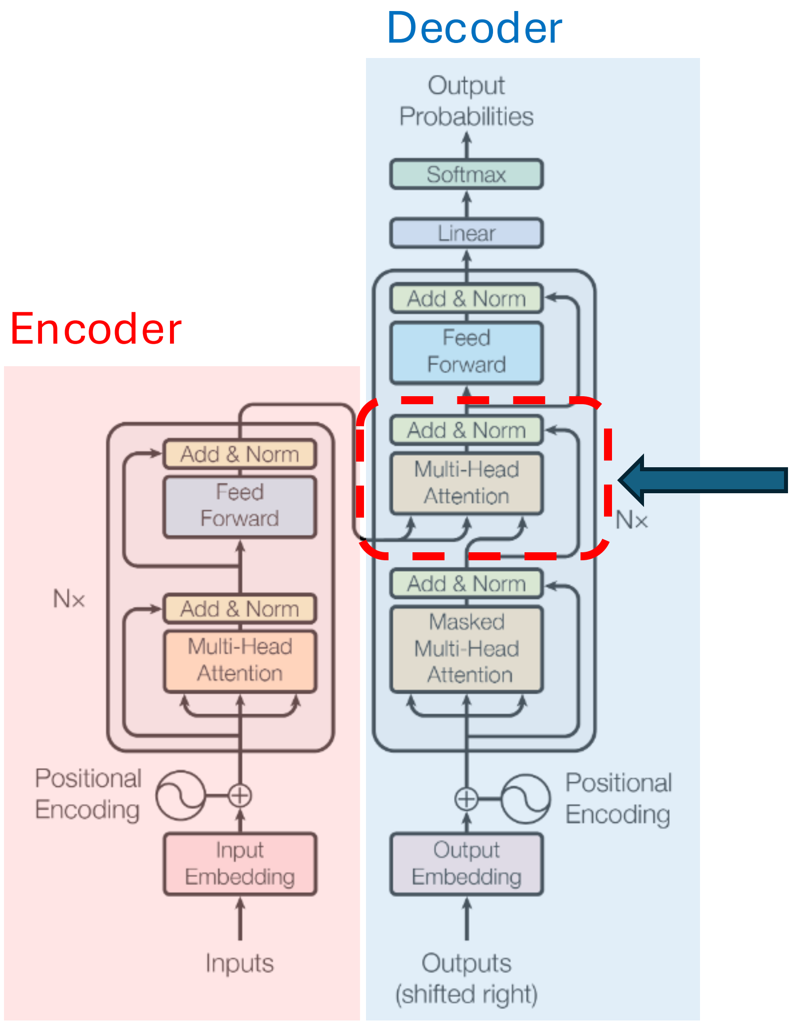
- Prompt with token string “<start> It takes great”
- Generate next token for the sequence by
  - picking most likely token
  - sample from the probability distribution
    - alternative *top-k* sampling to avoid picking from the long tail
  - beam search – select the most likely sentence rather than greedily pick

Encoder-Decoder Model

- Used for *machine translation*, which is a *sequence-to-sequence* task

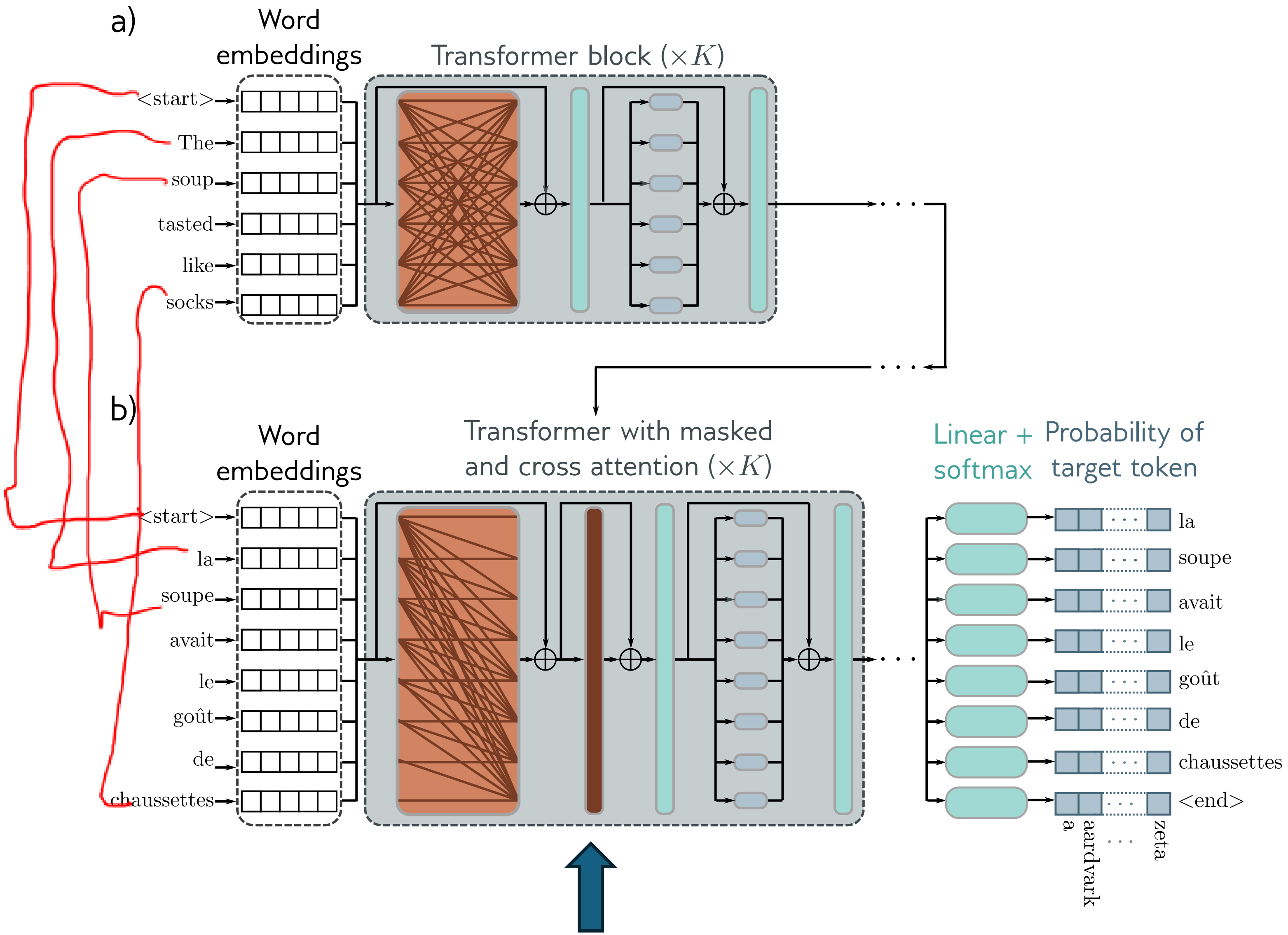
https://jalammar.github.io/illustrated-transformer/

Encoder Decoder Model



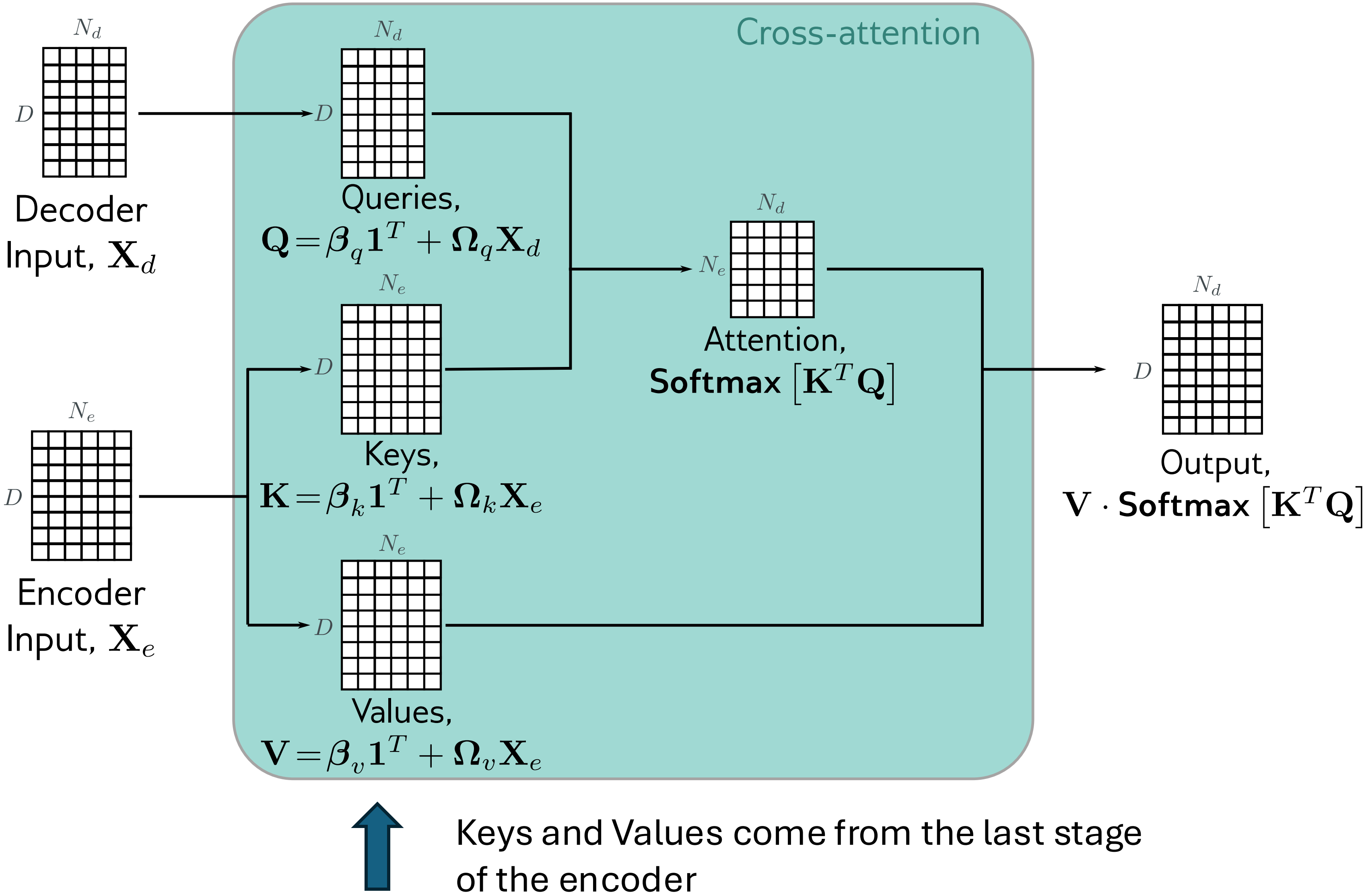
- The transformer layer in the decoder of the encoder-decoder model has an extra stage
- Attends to the input of the encoder with *cross attention* using Keys and Values from the output of the encoder
- Shown here on original diagram from “Attention is all you need” paper

Encoder Decoder Model



- Same view per UDL book

Cross-Attention



Keys and Values come from the last stage of the encoder

Any Questions?

???

Moving on

- Transformer recap
- What are tokens?
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What’s a Token?

A small chunk of text that we use to aid language modeling.

- Represents one or more bytes
- Input texts are greedily divided into tokens.
  - Longest prefix matching a token.
- Token set also constructed greedily.
  - Start with 256 possible bytes.
  - Then greedily pick the most common pairs of adjacent tokens.

Why Tokens?

Instead of…

- Bits - not enough semantics* and missing intrabyte positioning
- Bytes - not enough semantics* for Unicode
- Characters - too many of them if we try to support all languages
- Words - even more words than characters

whichever chosen

Remember:

- One-hot/Softmax tactic means we will have at least one output per possible output value, and many more parameters in practice.

Unicode Standard and UTF-8

- Unicode – *variable length* character encoding standard. currently defines 149,813 characters and 161 scripts, including emoji, symbols, etc.
- Unicode Codepoint – can represent up to $17 \times 2^{16} = 1{,}114{,}112$ entries. e.g. U+0000 – U+10FFFF in hexadecimal

  defined so far, not all used
- Unicode Transformation Standard (e.g. UTF-8) – is a *variable length encoding* using one to four bytes
  - First 128 chars same as ASCII

Code point ↔ UTF-8 conversion

First code point	Last code point	Byte 1	Byte 2	Byte 3	Byte 4	
U+0000	U+007F	0xxxxxxx				Covers ASCII
U+0080	U+07FF	110xxxxx	10xxxxxx			Covers remainder of almost all Latin-script alphabets
U+0800	U+FFFF	1110xxxx	10xxxxxx	10xxxxxx		Basic Multilingual Plane including Chinese, Japanese and Korean characters
U+010000	[b]U+10FFFF	11110xxx	10xxxxxx	10xxxxxx	10xxxxxx	Emoji, historic scripts, math symbols

https://en.wikipedia.org/wiki/Unicode
https://en.wikipedia.org/wiki/UTF-8

variable length

Example Tokens

```python
import tiktoken
```

```python
[6] enc = tiktoken.encoding_for_model("gpt-4o")
```

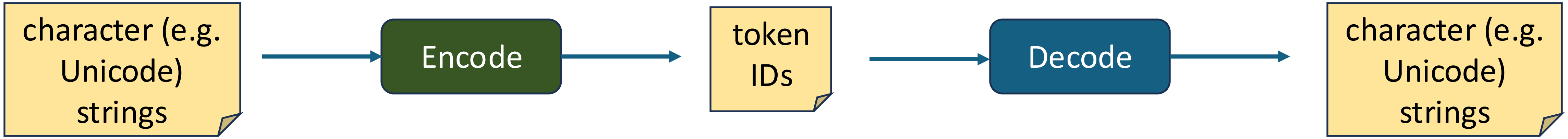
```python
for i in range(1024):
    d = enc.decode([i])
    if len(d) >= 4:
        print(i, enc.decode([i]))
```

```
257
269
290  the
309
326  and
352
387 ation
395  for
406  con
408
440  pro
447 port
452  com
475 ction
481  you
483  with
484  that
495  this
506
508 ment
518 ----
529 turn
530
553  are
561 import
562 able
583 ight
584 ublic
591  from
595 ****
600 tring
620  new
622  return
623  The
625  not
626
634  your
661  que
665  can
673  was
677  int
679  have
686  par
694  res
699
700 form
717  get
722  all
728 ject
731  des
735 alue
738  will
740 ();

744  class
751  public
756 ions
758  }


763 --------
766 ance
767 ould
773 ient
775 .get
```

Tokenizer



Tokenizer chooses input “units”, e.g. words, sub-words, characters via *tokenizer training*

which tokens do we want?

In tokenizer training, commonly occurring substrings are greedily merged based on their frequency, starting with character pairs

Tokenization Issues

“A lot of the issues that may look like issues with the neural network architecture actually trace back to tokenization. Here are just a few examples” – Andrej Karpathy

- Why can't LLM spell words? Tokenization.
- Why can't LLM do super simple string processing tasks like reversing a string? Tokenization.
- Why is LLM worse at non-English languages (e.g. Japanese)? Tokenization.
- Why is LLM bad at simple arithmetic? Tokenization.
- Why did GPT-2 have more than necessary trouble coding in Python? Tokenization.
- Why did my LLM abruptly halt when it sees the string "<|endoftext|>"? Tokenization.
- What is this weird warning I get about a "trailing whitespace"? Tokenization.
- Why did the LLM break if I ask it about "SolidGoldMagikarp"? Tokenization.
- Why should I prefer to use YAML over JSON with LLMs? Tokenization.
- Why is LLM not actually end-to-end language modeling? Tokenization.
- What is the real root of suffering? Tokenization.

https://github.com/karpathy/minbpe/blob/master/lecture.md

Any Questions?

???

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Tokenization Matters

From the gpt-4o announcement,

“It matches GPT-4 Turbo performance on text in English and code, with significant improvement on text in non-English languages, while also being much faster and 50% cheaper in the API.”

Gains were from increasing the number of tokens in the updated tokenizer.

https://openai.com/index/hello-gpt-4o/

Russian 1.7x fewer tokens (from 39 to 23)	Привет, меня зовут GPT-4o. Я — новая языковая модель, приятно познакомиться!
Korean 1.7x fewer tokens (from 45 to 27)	안녕하세요, 제 이름은 GPT-4o입니다. 저는 새로운 유형의 언어 모델입니다, 만나서 반갑습니다!
Vietnamese 1.5x fewer tokens (from 46 to 30)	Xin chào, tên tôi là GPT-4o. Tôi là một loại mô hình ngôn ngữ mới, rất vui được gặp bạn!
Chinese 1.4x fewer tokens (from 34 to 24)	你好，我的名字是GPT-4o。我是一种新型的语言模型，很高兴见到你!
Japanese 1.4x fewer tokens (from 37 to 26)	こんにちは、私の名前はGPT-4oです。私は新しいタイプの言語モデルです。初めまして！

Tokenizer

Two common tokenizers:

- Byte Pair Encoding (BPE) – Used by OpenAI GPT2, GPT4, etc.
  - The BPE algorithm is "byte-level" because it runs on UTF-8 encoded strings.
  - This algorithm was popularized for LLMs by the GPT-2 paper and the associated GPT-2 code release from OpenAI. Sennrich et al. 2015 is cited as the original reference for the use of BPE in NLP applications. Today, all modern LLMs (e.g. GPT, Llama, Mistral) use this algorithm to train their tokenizers.*
- sentencepiece
  - (e.g. Llama, Mistral) use sentencepiece instead. Primary difference being that sentencepiece runs BPE directly on Unicode code points instead of on UTF-8 encoded bytes.

* https://github.com/karpathy/minbpe/tree/master

BPE Pseudocode

almost always bytes, not full characters

Initialize vocabulary with individual characters in the text and their frequencies

While desired vocabulary size not reached:

this is width of token prediction output

- Identify the most frequent pair of adjacent tokens/characters in the vocabulary
- Merge this pair to form a new token
- Update the vocabulary with this new token
- Recalculate frequencies of all tokens including the new token

Return the final vocabulary

Enforce a Token Split Pattern

```
GPT2_SPLIT_PATTERN = r"""'(?:[sdmt]|ll|ve|re)| ?\p{L}+| ?\p{N}+| ?[^\s\p{L}\p{N}]+|\s+(?!\S)|\s+"""
```

```
GPT4_SPLIT_PATTERN = r"""'(?i:[sdmt]|ll|ve|re)|[^\r\n\p{L}\p{N}]?+\p{L}+|\p{N}{1,3}|
?[^\s\p{L}\p{N}]++[\r\n]*|\s*[\r\n]|\s+(?!\S)|\s+"""
```

- Do not allow tokens to merge across certain characters or patterns
- Common contraction endings: ‘ll, ‘ve, ‘re
- Match words with a leading space
- Match numeric sequences
- carriage returns, new lines

GPT4 Tokenizer

cl100k_base is the GPT4 tokenizer

Tiktokenizer

cl100k_base

```
a sailor went to sea sea sea
to see what he could see see see
but all that he could see see see
was the bottom of the deep blue sea sea sea
```

Token count

36

```
a·sailor·went·to·sea·sea·sea\n
to·see·what·he·could·see·see·see\n
but·all·that·he·could·see·see·see\n
was·the·bottom·of·the·deep·blue·sea·sea·sea
```

```
[64, 93637, 4024, 311, 9581, 9581, 9581, 198, 998, 1518, 1148, 568, 1436, 1518, 1518, 1518, 198, 8248, 682, 430, 568, 1436, 1518, 1518, 1518, 198, 16514, 279, 5740, 315, 279, 5655, 6437, 9581, 9581, 9581]
```

☑ Show whitespace

https://tiktokenizer.vercel.app/

GPT2 Tokenizer

Tiktokenizer

```
class Tokenizer:
    """Base class for Tokenizers"""

    def __init__(self):
        # default: vocab size of 256 (all bytes), no merges,
no patterns
        self.merges = {} # (int, int) -> int
        self.pattern = "" # str
        self.special_tokens = {} # str -> int, e.g.
{'<|endoftext|>': 100257}
        self.vocab = self._build_vocab() # int -> bytes
```

gpt2

Token count
146

```
class·Tokenizer:\n
····"""Base·class·for·Tokenizers"""\n
\n
····def·__init__(self):\n
········#·default:·vocab·size·of·256·(all·bytes),·no·m
erges,·no·patterns\n
········self.merges·=·{}·#·(int,·int)·->·int\n
········self.pattern·=·""·#·str\n
········self.special_tokens·=·{}·#·str·->·int,·e.g.·
{'<|endoftext|>':·100257}\n
········self.vocab·=·self._build_vocab()·#·int·->·byte
s
```

```
[4871, 29130, 7509, 25, 198, 220, 220, 220, 37227, 148
81, 1398, 329, 29130, 11341, 37811, 628, 220, 220, 22
0, 825, 11593, 15003, 834, 7, 944, 2599, 198, 220, 22
0, 220, 220, 220, 220, 220, 1303, 4277, 25, 12776, 39
7, 2546, 286, 17759, 357, 439, 9881, 828, 645, 4017, 3
212, 11, 645, 7572, 198, 220, 220, 220, 220, 220, 220,
220, 2116, 13, 647, 3212, 796, 23884, 1303, 357, 600,
11, 493, 8, 4613, 493, 198, 220, 220, 220, 220, 220, 2
20, 220, 2116, 13, 33279, 796, 13538, 1303, 965, 198,
220, 220, 220, 220, 220, 220, 220, 2116, 13, 20887, 6
2, 83, 482, 641, 796, 23884, 1303, 965, 4613, 493, 11,
304, 13, 70, 13, 1391, 6, 50256, 10354, 1802, 28676, 9
2, 198, 220, 220, 220, 220, 220, 220, 220, 2116, 13, 1
8893, 397, 796, 2116, 13557, 11249, 62, 18893, 397, 34
19, 1303, 493, 4613, 9881]
```

✓ Show whitespace

You can see some issues with the GPT2 tokenizer with respect to python code

https://tiktokenizer.vercel.app/

GPT4 Tokenizer

Tiktokenizer

cl100k_base

```
class Tokenizer:
    """Base class for Tokenizers"""

    def __init__(self):
        # default: vocab size of 256 (all bytes), no merges,
no patterns
        self.merges = {} # (int, int) -> int
        self.pattern = "" # str
        self.special_tokens = {} # str -> int, e.g.
{'<|endoftext|>': 100257}
        self.vocab = self._build_vocab() # int -> bytes
```

Token count
96

```
class·Tokenizer:\n
····"""Base·class·for·Tokenizers"""\n
\n
····def·__init__(self):\n
········#·default:·vocab·size·of·256·(all·bytes),·no·m
erges,·no·patterns\n
········self.merges·=·{}·#·(int,·int)·->·int\n
········self.pattern·=·""·#·str\n
········self.special_tokens·=·{}·#·str·->·int,·e.g.·
{'<|endoftext|>':·100257}\n
········self.vocab·=·self._build_vocab()·#·int·->·byte
s
```

```
[1058, 9857, 3213, 512, 262, 4304, 4066, 538, 369, 985
7, 12509, 15425, 262, 711, 1328, 2381, 3889, 726, 997,
286, 674, 1670, 25, 24757, 1404, 315, 220, 4146, 320,
543, 5943, 705, 912, 82053, 11, 912, 12912, 198, 286,
659, 749, 2431, 288, 284, 4792, 674, 320, 396, 11, 52
8, 8, 1492, 528, 198, 286, 659, 40209, 284, 1621, 674,
610, 198, 286, 659, 64308, 29938, 284, 4792, 674, 610,
1492, 528, 11, 384, 1326, 13, 5473, 100257, 1232, 220,
1041, 15574, 534, 286, 659, 78557, 284, 659, 1462, 595
7, 53923, 368, 674, 528, 1492, 5943]
```

☑ Show whitespace

Issues are improved with GPT4 tokenizer

https://tiktokenizer.vercel.app/

Byte Pair Encoding (BPE) Example

Minimal starting vocabulary of subset of lower case latin alphabet and space `_`.

a)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	s	a	t	o	h	l	u	b	d	w	c	f	i	m	n	p	r
33	28	15	12	11	8	6	6	4	3	3	3	2	1	1	1	1	1	1

Byte Pair Encoding (BPE) Example

Find the most frequent pair of adjacent tokens, `se`, in this case and form new token.

sea or see

a)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	s	a	t	o	h	l	u	b	d	w	c	f	i	m	n	p	r
33	28	15	12	11	8	6	6	4	3	3	3	2	1	1	1	1	1	1

b)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	se	a	t	o	h	l	u	b	d	w	c	s	f	i	m	n	p	r
33	15	13	12	11	8	6	6	4	3	3	3	2	2	1	1	1	1	1	1

Byte Pair Encoding (BPE) Example

a)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	s	a	t	o	h	l	u	b	d	w	c	f	i	m	n	p	r
33	28	15	12	11	8	6	6	4	3	3	3	2	1	1	1	1	1	1

b)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	se	a	t	o	h	l	u	b	d	w	c	s	f	i	m	n	p	r
33	15	13	12	11	8	6	6	4	3	3	3	2	2	1	1	1	1	1	1

c)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	se	a	e_	t	o	h	l	u	b	d	e	w	c	s	f	i	m	n	p	r
21	13	12	12	11	8	6	6	4	3	3	3	3	2	2	1	1	1	1	1	1

Next most frequent pair of tokens is `e_`

Byte Pair Encoding (BPE) Example

a)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	s	a	t	o	h	l	u	b	d	w	c	f	i	m	n	p	r
33	28	15	12	11	8	6	6	4	3	3	3	2	1	1	1	1	1	1

b)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	se	a	t	o	h	l	u	b	d	w	c	s	f	i	m	n	p	r
33	15	13	12	11	8	6	6	4	3	3	3	2	2	1	1	1	1	1	1

c)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	se	a	e_	t	o	h	l	u	b	d	e	w	c	s	f	i	m	n	p	r
21	13	12	12	11	8	6	6	4	3	3	3	3	2	2	1	1	1	1	1	1

⋮ ⋮

Continue until you hit your vocabulary size limit.

d)

see_	sea_	e	b	l	w	a	could_	hat_	he_	o	t	t_	the_	to_	u	a_	d	f	m	n	p	s	sailor_	to
7	6	4	3	3	3	3	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1

2x 4 char tokens that are common

worth it?

Byte Pair Encoding (BPE) Example

a)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	s	a	t	o	h	l	u	b	d	w	c	f	i	m	n	p	r
33	28	15	12	11	8	6	6	4	3	3	3	2	1	1	1	1	1	1

b)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	se	a	t	o	h	l	u	b	d	w	c	s	f	i	m	n	p	r
33	15	13	12	11	8	6	6	4	3	3	3	2	2	1	1	1	1	1	1

c)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	se	a	e_	t	o	h	l	u	b	d	e	w	c	s	f	i	m	n	p	r
21	13	12	12	11	8	6	6	4	3	3	3	3	2	2	1	1	1	1	1	1

⋮ ⋮

d)

see_	sea_	e	b	l	w	a	could_	hat_	he_	o	t	t_	the_	to_	u	a_	d	f	m	n	p	s	sailor_	to
7	6	4	3	3	3	3	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1

⋮ ⋮ ⋮

e)

see_	sea_	could_	he_	the_	a_	all_	blue_	bottom_	but_	deep_	of_	sailor_	that_	to_	was_	went_	what_
7	6	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1

a)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	s	a	t	o	h	l	u	b	d	w	c	f	i	m	n	p	r
33	28	15	12	11	8	6	6	4	3	3	3	2	1	1	1	1	1	1

b)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	e	se	a	t	o	h	l	u	b	d	w	c	s	f	i	m	n	p	r
33	15	13	12	11	8	6	6	4	3	3	3	2	2	1	1	1	1	1	1

c)

a_sailor_went_to_sea_sea_sea_
to_see_what_he_could_see_see_see_
but_all_that_he_could_see_see_see_
was_the_bottom_of_the_deep_blue_sea_sea_sea_

_	se	a	e_	t	o	h	l	u	b	d	e	w	c	s	f	i	m	n	p	r
21	13	12	12	11	8	6	6	4	3	3	3	3	2	2	1	1	1	1	1	1

⋮ ⋮

d)

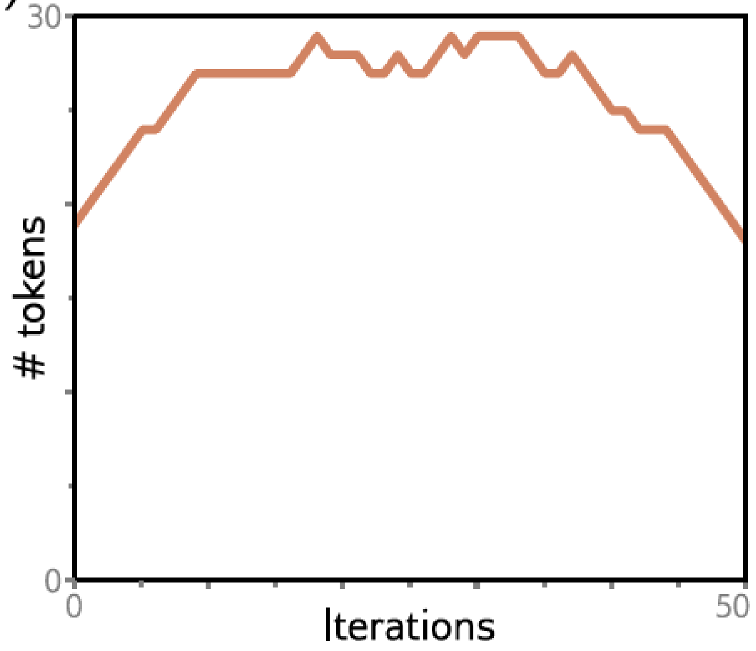
see_	sea_	e	b	l	w	a	could_	hat_	he_	o	t	t_	the_	to_	u	a_	d	f	m	n	p	s	sailor_	to
7	6	4	3	3	3	3	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1

⋮ ⋮ ⋮

e)

see_	sea_	could_	he_	the_	a_	all_	blue_	bottom_	but_	deep_	of_	sailor_	that_	to_	was_	went_	what_
7	6	2	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1

f)



Generally # of tokens increases and then starts decreasing after continuing to merge tokens

NLP Preprocessing Pipeline

Transformers don’t work on character string directly, but rather on vectors.

The character strings must be converted to vectors

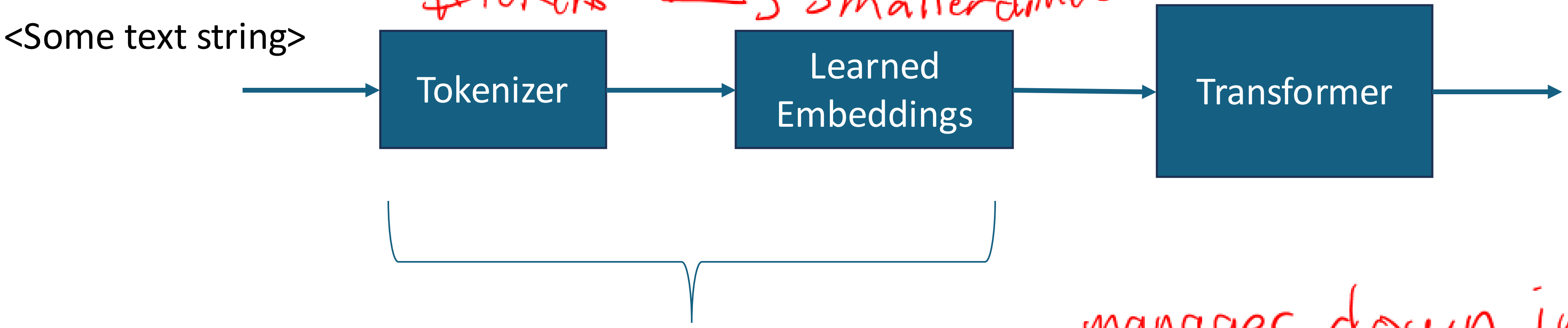
one hot

#tokens

like one hot encoding?

more dense

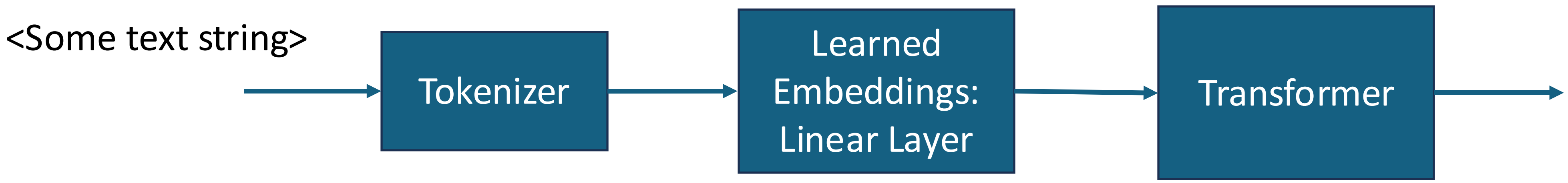
→ smaller dimension



Preprocessing: Tokenization and Embedding

manages down input cost of token one hot encoding

Learned Embeddings



- After the tokenizer, you have an updated "vocabulary" indexed by token ID
- Next step is to translate the token into an embedding vector
- Translation is done via a linear layer which is typically learned with the rest of the transformer model

tokens

```
self.embedding = nn.Embedding(vocab_size, embedding_dim)
```

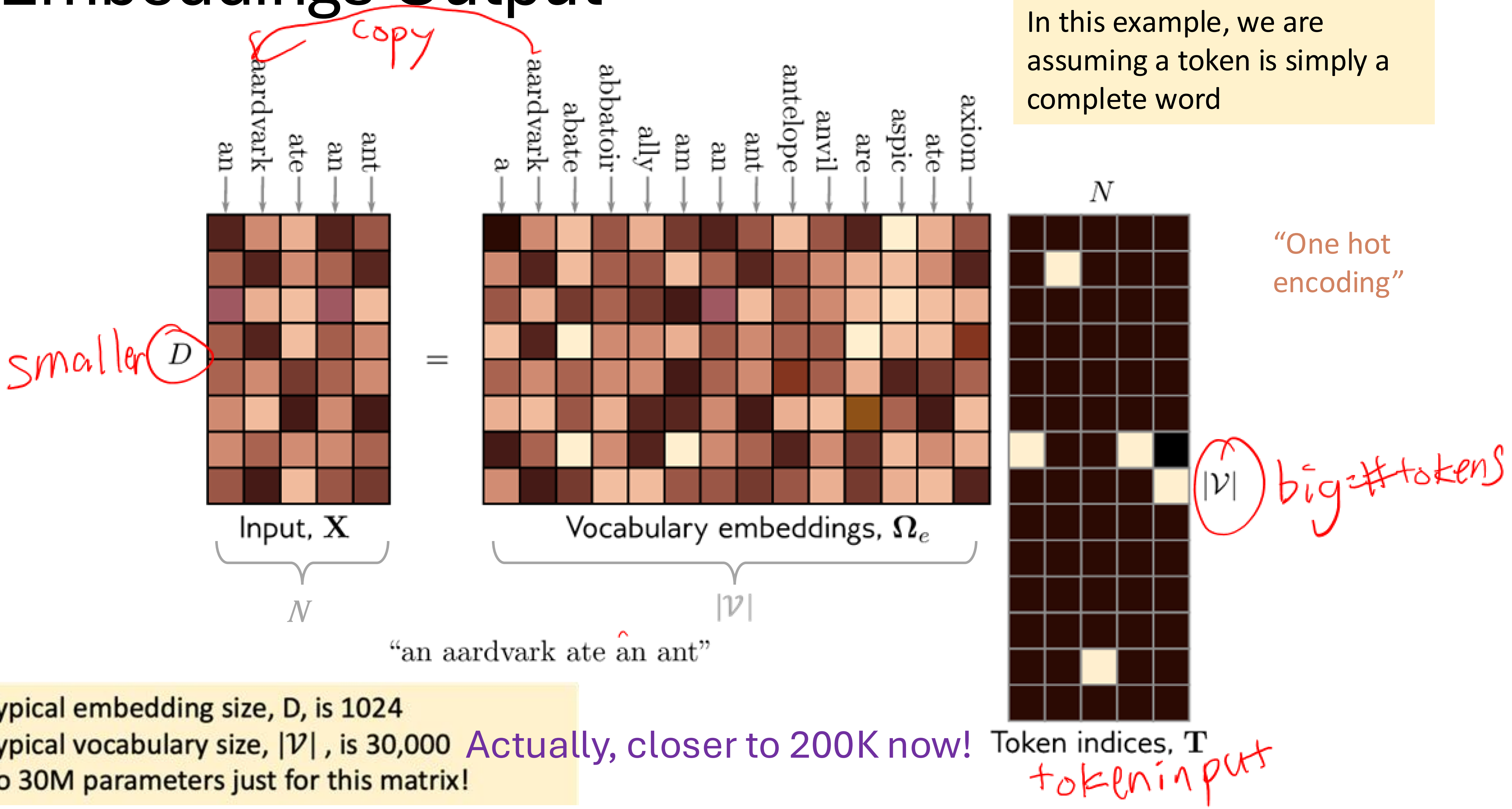
big small

- Special layer definition, likely to exploit sparsity of input

b/c onehot tokens

Embeddings Output

In this example, we are assuming a token is simply a complete word



"One hot encoding"

- Typical embedding size, D, is 1024
- Typical vocabulary size, $|\mathcal{V}|$, is 30,000 Actually, closer to 200K now!
- So 30M parameters just for this matrix!

Any Questions?

???

Moving on

- Transformer recap
- What are tokens?
- Tokenization and word embedding
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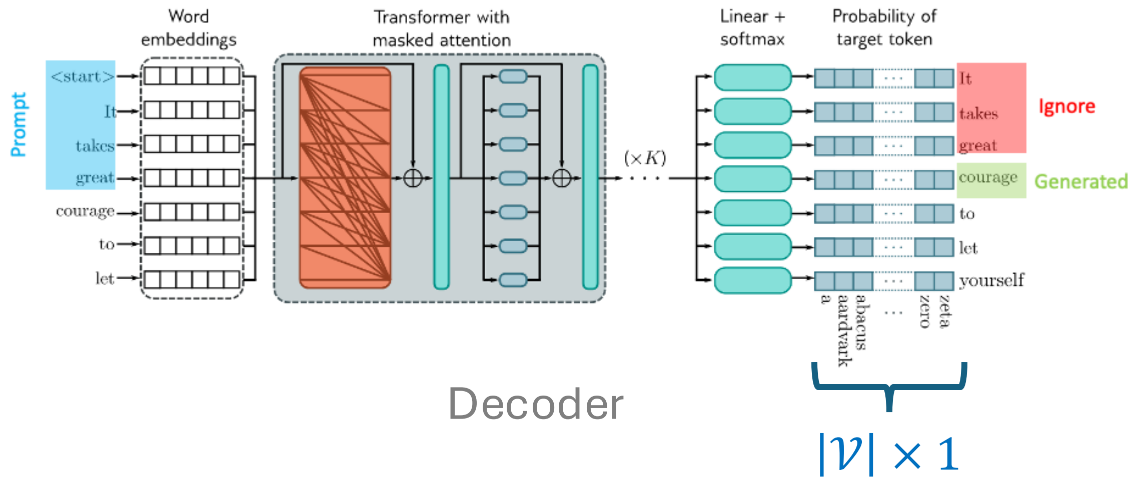
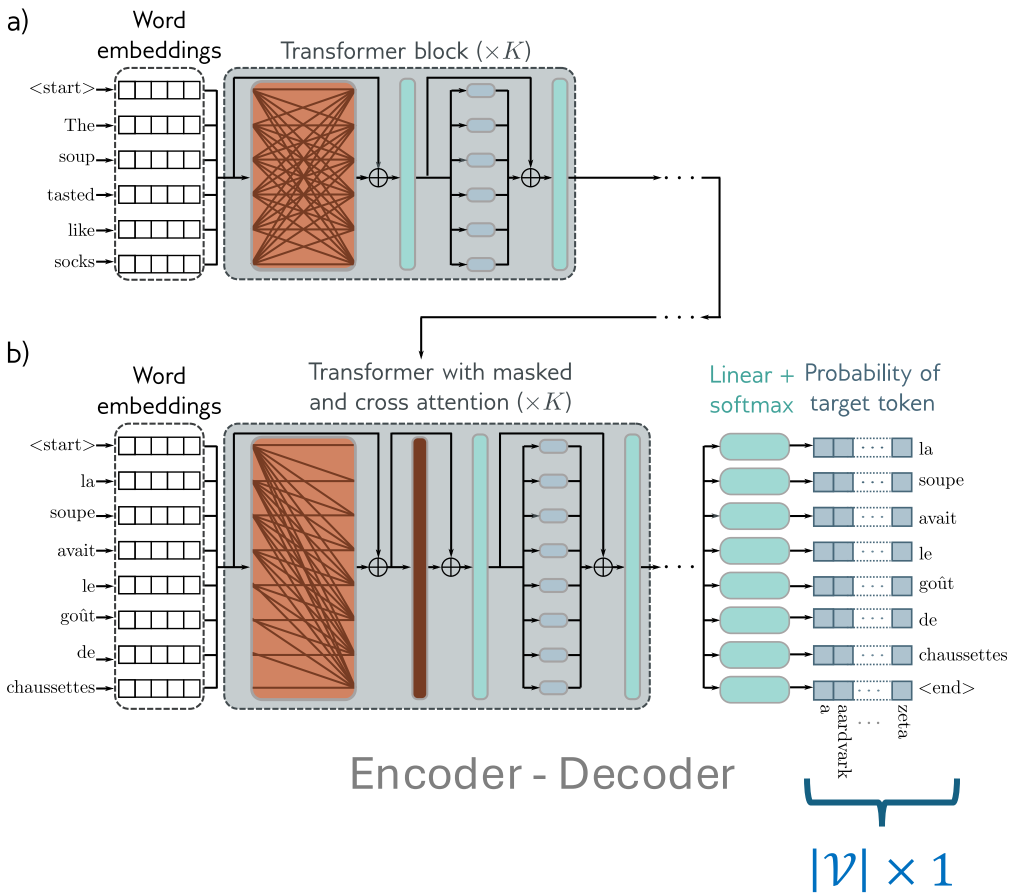
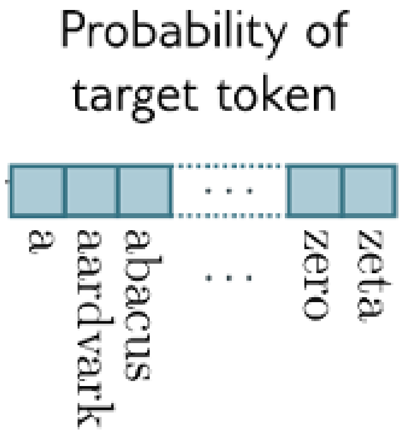
Next Token Selection





- Recall: output is a $|\mathcal{V}| \times 1$ vector of probabilities
- How should we pick the next token?
- Trade off between accuracy and diversity

Next Token Selection



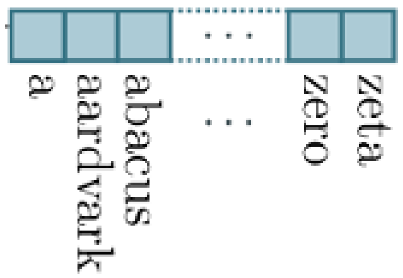
Recall: output is a $|\mathcal{V}| \times 1$ vector of probabilities

Selectin methods:

- Greedy selection
- Top-K
- Nucleus
- Temperature
- Beam search

Next Token Selection – Greedy

Probability of target token



Pick most likely token (greedy)

Simple to implement. Just take the max().

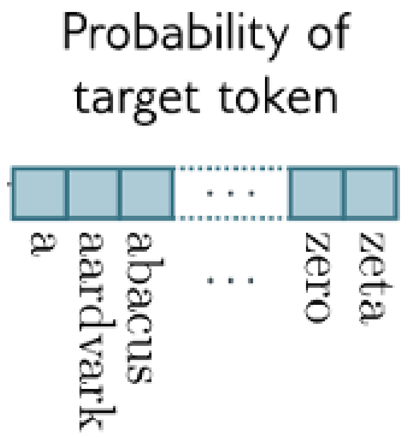
$$\hat{y}_t = \underset{w \in \mathcal{V}}{\operatorname{argmax}} \left[Pr(y_t = w | \hat{\mathbf{y}}_{<t}, \mathbf{x}, \phi) \right]$$

```
# in PyTorch
outputs = model(inputs)
value, index = outputs.max(1)
```

Might pick first token y_0, but then there is no y_1 where $\mathrm{Pr}(y_1 | \, y_0)$ is high.

Result is generic and predictable. Same output for a given input context. repeatable

Next Token Selection -- Sampling



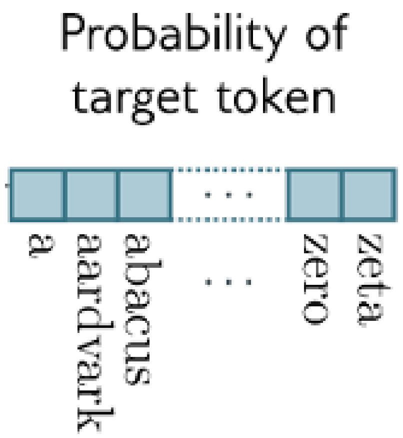
Sample from the probability distribution

Get a bit more diversity in the output

Will occasionally sample from the long tail of the distribution, producing some unlikely word combinations.

But real text does have unlikely words too.

Next Token Selection – Top *K* Sampling



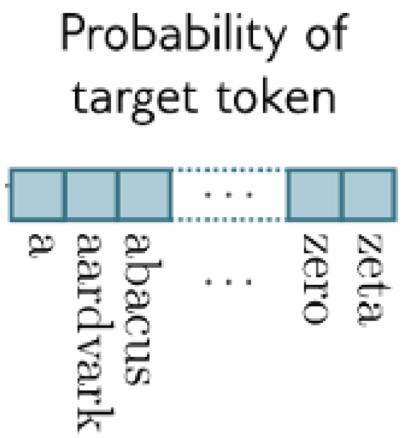
1. Generate the probability vector as usual
2. Sort tokens by likelihood
3. Discard all but top *k* most probable words
4. Renormalize the probabilities to be valid probability distribution (e.g. sum to 1)
5. Sample from the new distribution

Diversifies word selection.

Depends on the distribution. Could be low variance, reducing diversity.

If only one good choice, forced k-1 tokens would be funky

Next Token Selection – Nucleus Sampling



Instead of keeping top-*k*, keep the top *p* percent of the probability mass.

Choose from the smallest set from the vocabulary such that

$$\sum_{w \in V^{(p)}} P(w|\mathbf{w}_{<t}) \geq p.$$

Diversifies word selection with less dependence on nature of distribution.

Depends on the distribution. Could be low variance, reducing diversity.

Next Token Selection - Temperature

- Before applying softmax to calculate probabilities, divide the logit outputs by a temperature T. new param

normal

$$\text{softmax}_\text{i}(\mathbf{z}) = \frac{e^{z_i}}{\sum_j e^{z_j}}$$

temperature

$$\text{softmax}_\text{i}(\mathbf{z}, T) = \frac{e^{z_i/T}}{\sum_j e^{z_j/T}}$$

- What happens as $T \to \infty$?

exponents $\to 0$

$e^{z_i/T} \to 1$

$P_i = \frac{1}{\#\text{tokens}}$

- What happens as $T \to 0$?

$\to$ greedy sampling,

Ø works in serious implementation

- Offered in most LLM interfaces.

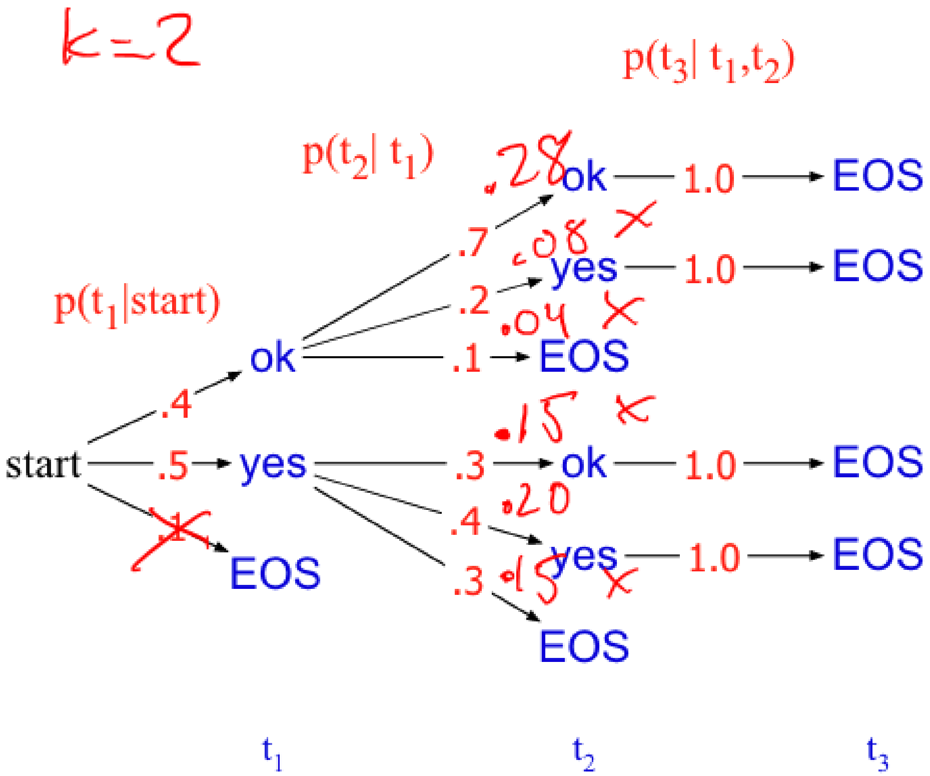
Next Token Selection – Beam Search

Commonly used in *machine translation*

Maintain multiple output choices and then choose best combinations later via tree search

V = {yes, ok, <eos>}

We want to maximize $p(t_1, t_2, t_3)$.



Greedy: $0.5 \times 0.4 \times 1.0 = 0.20$

Optimal: $0.4 \times 0.7 \times 1.0 = 0.28$

TLDR: keep best k paths at each level of tree.
Less popular as models have gotten bigger.

D. Jurafsky and J. H. Martin, *Speech and Language Processing*. 2024.
https://web.stanford.edu/~jurafsky/slpdraft/

Dummy’s Guide to LLM Sampling

- https://rentry.co/samplers

Dummy's Guide to Modern LLM Sampling

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???

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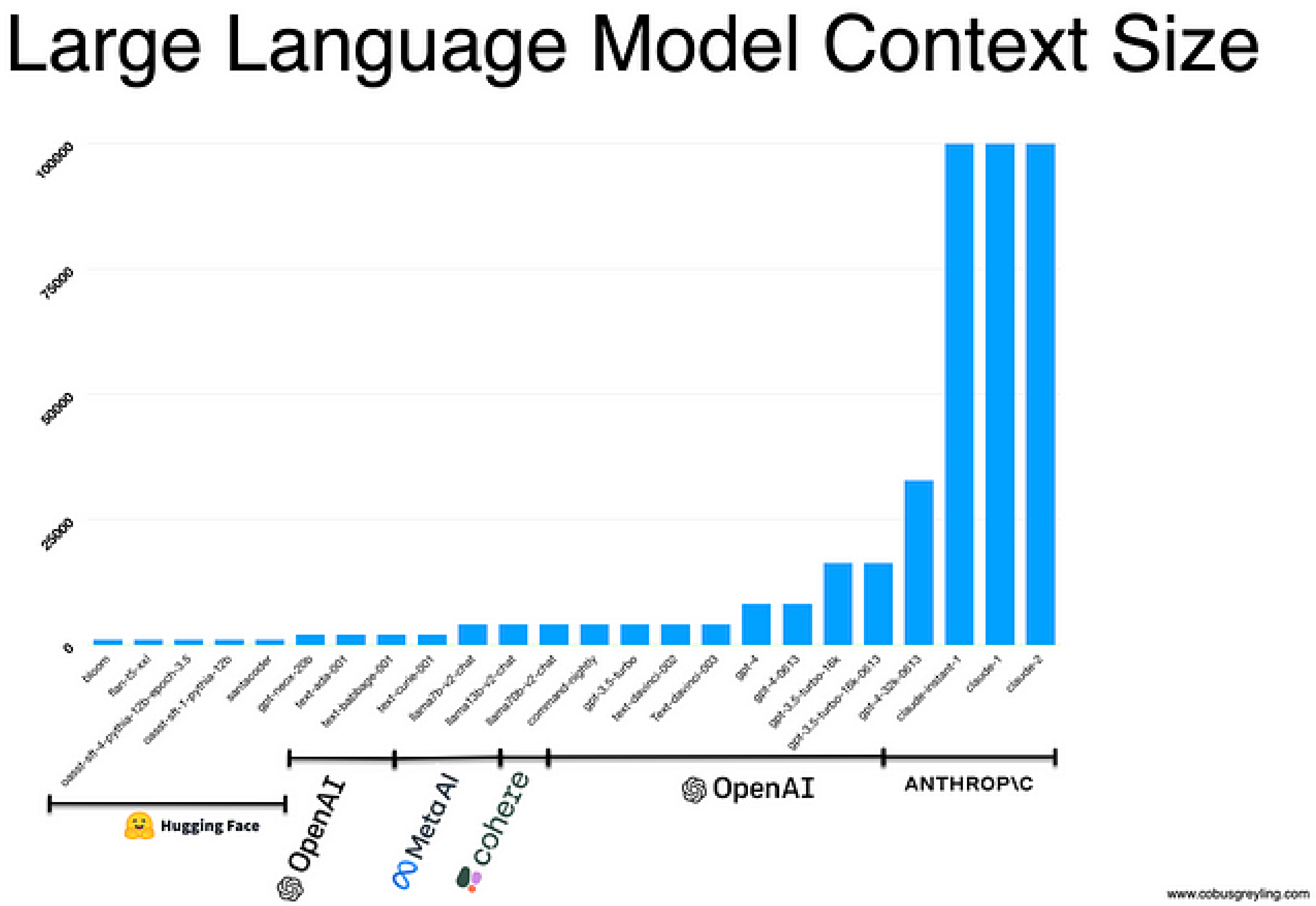
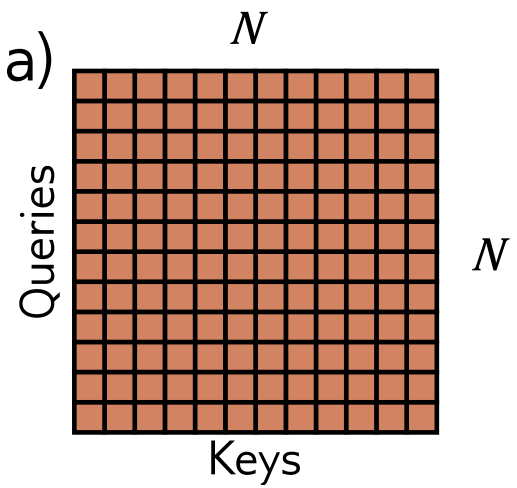
Context Length of LLMs

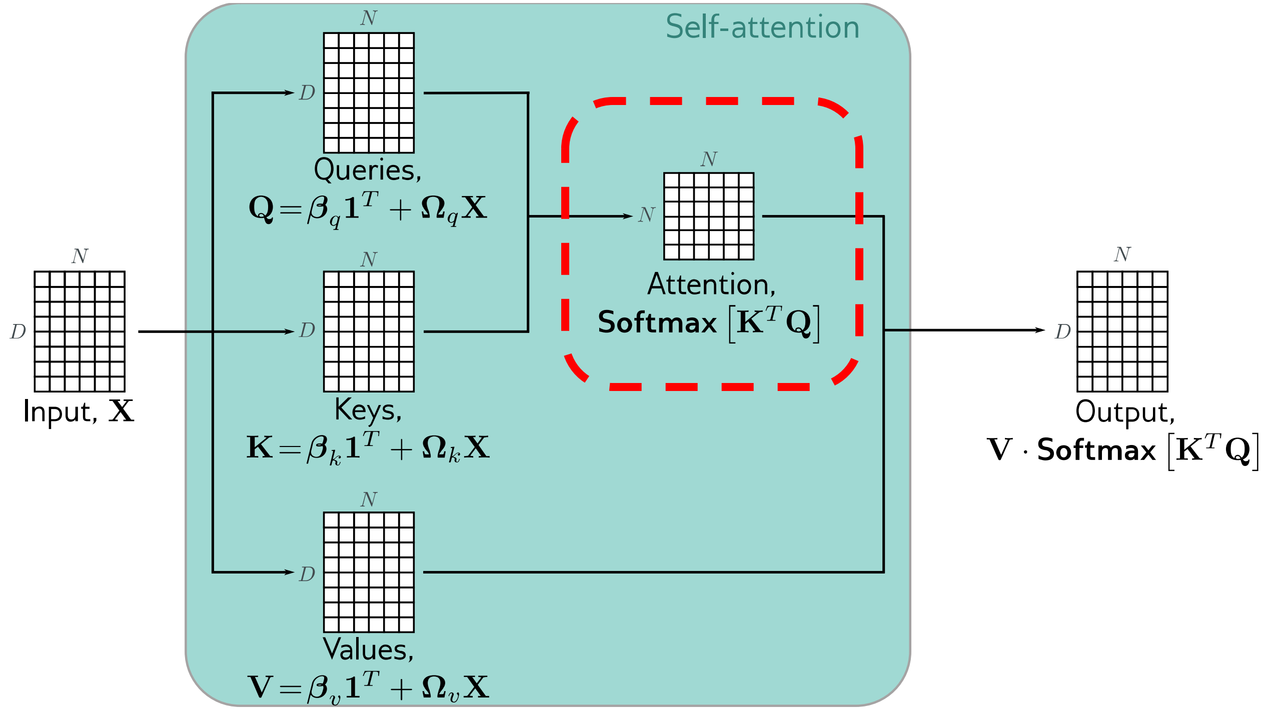
Model	Context Length
Llama 2	32K
GPT4	32K
GPT-4 Turbo, Llama 3.1	128K
Claude 3.5 Sonnet	200K
Google Gemini 1.5 Pro	Millions



https://cobusgreyling.medium.com/rag-llm-context-size-6728a2f44beb

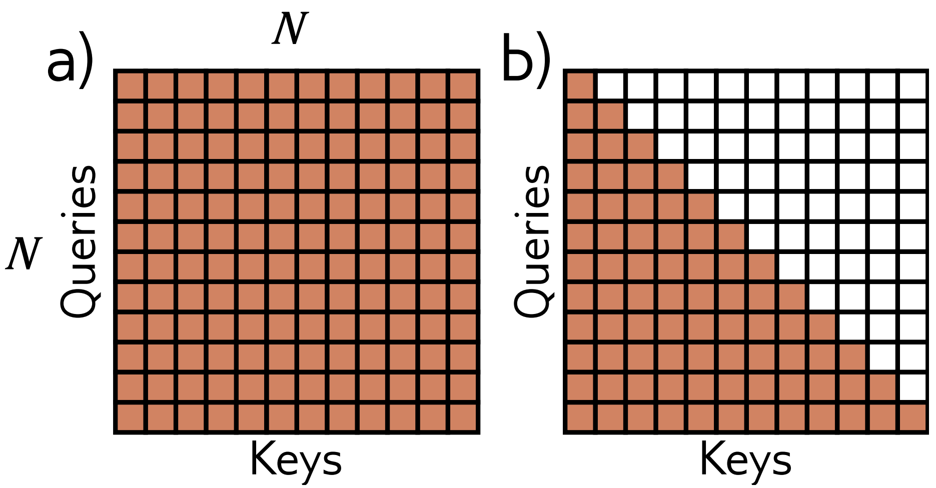
Attention Matrix

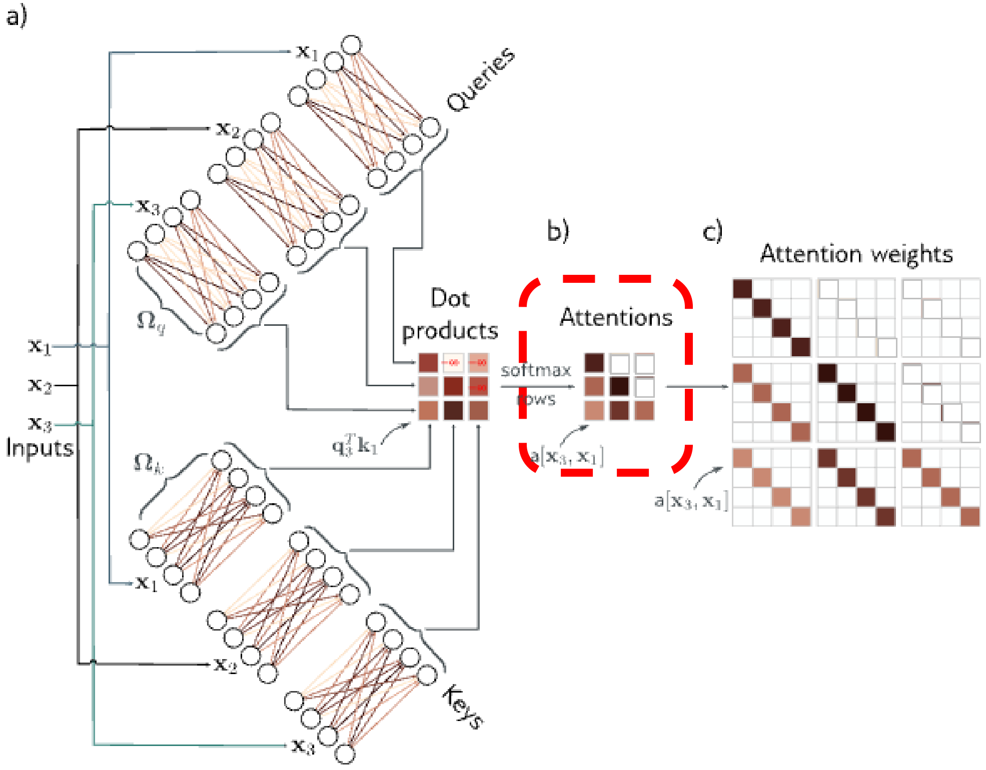




Scales quadratically with sequence length N, e.g. N^2.

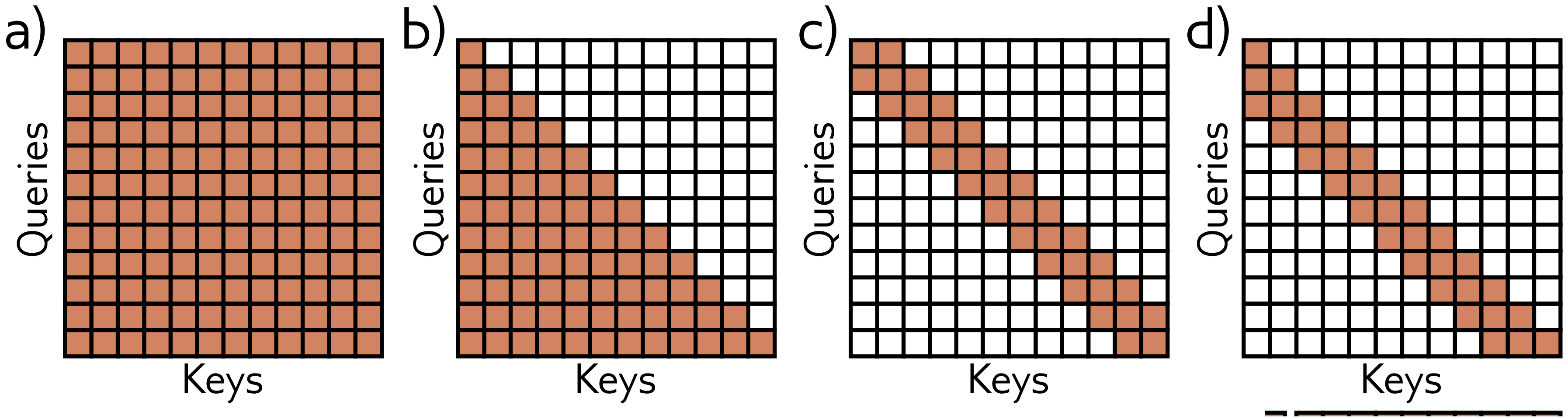
Masked Attention





~1/2 the interactions but still scales quadratically

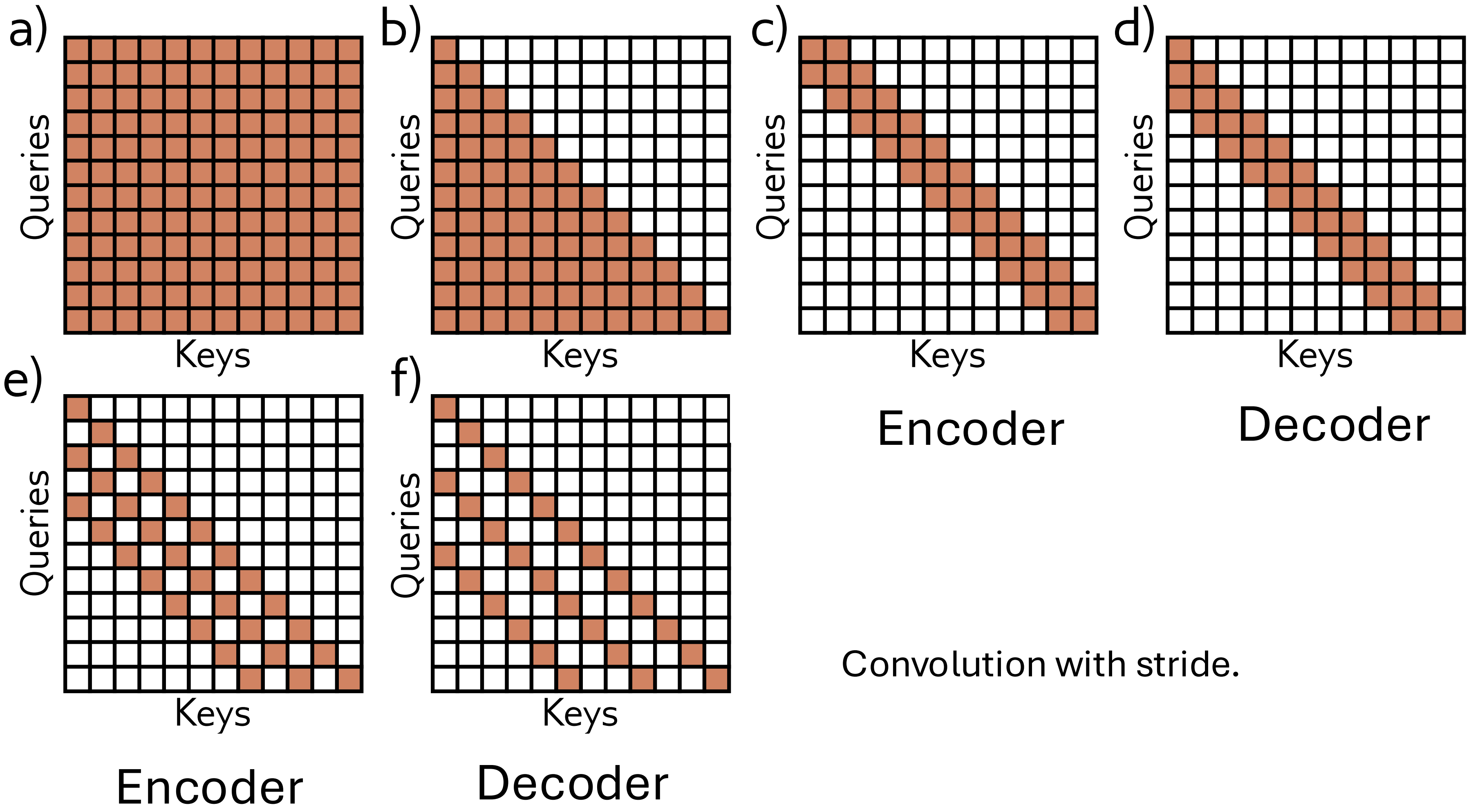
Use Convolutional Structure in Attention



Encoder
(non-causal)

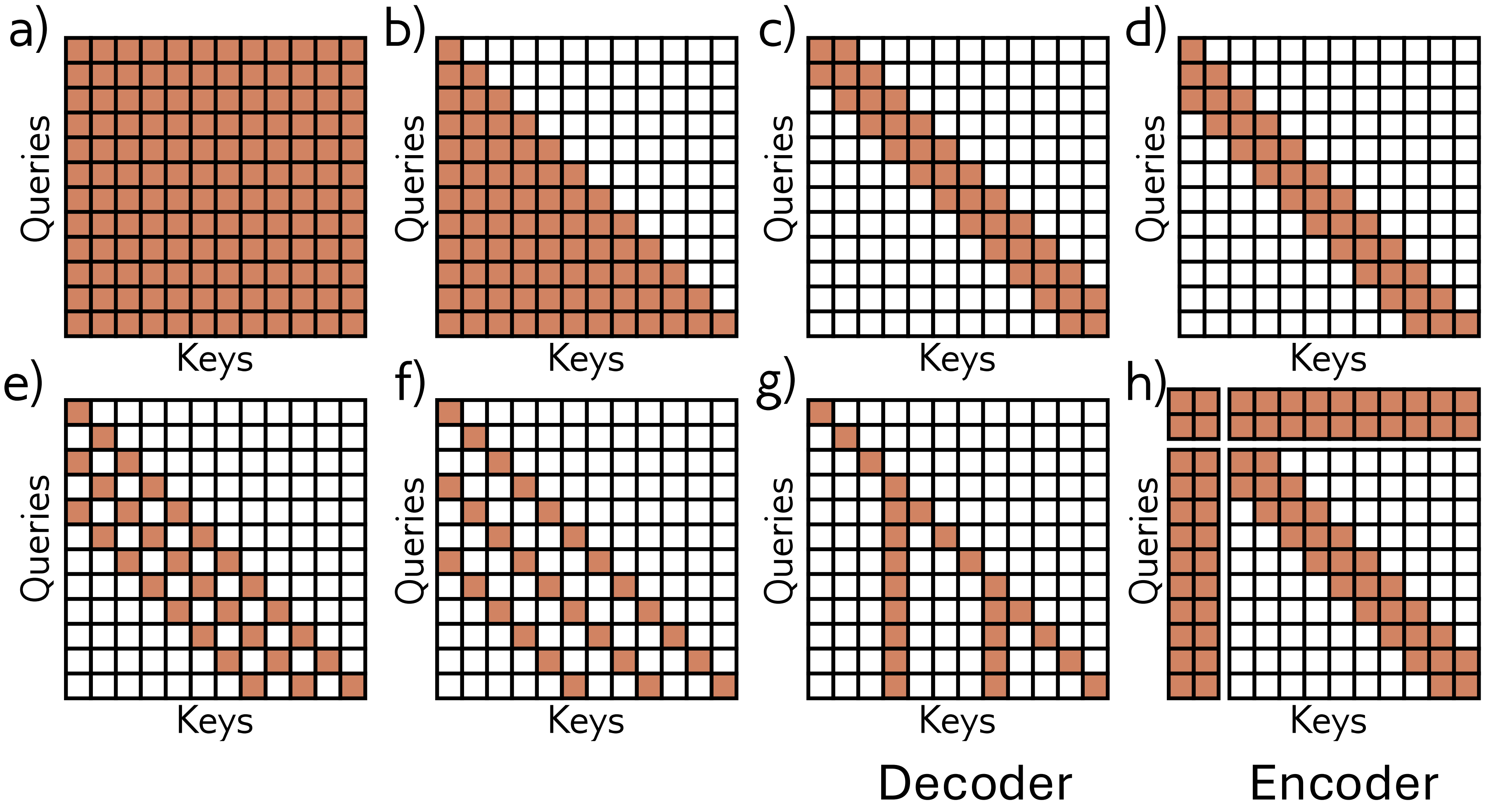
Decoder
(causal)

Dilated Convolutional Structures



Convolution with stride.

Have some tokens interact globally



Many of Attempts at Sub-Quadratic Attention

- AFAIK none in state-the-art-models
    - Many published papers claiming linear or $n \log n$ scaling with comparable performance, but none demonstrated at the same scale.
        - 10-20B parameters vs 500B parameters.

- Many practical speedups for leaner quadratic attention.
    - FlashAttention is popular. Same calculations but optimized to be IO-aware.

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???

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