BOSTON
UNIVERSITY

Deep Learning for Data Science
DS 542

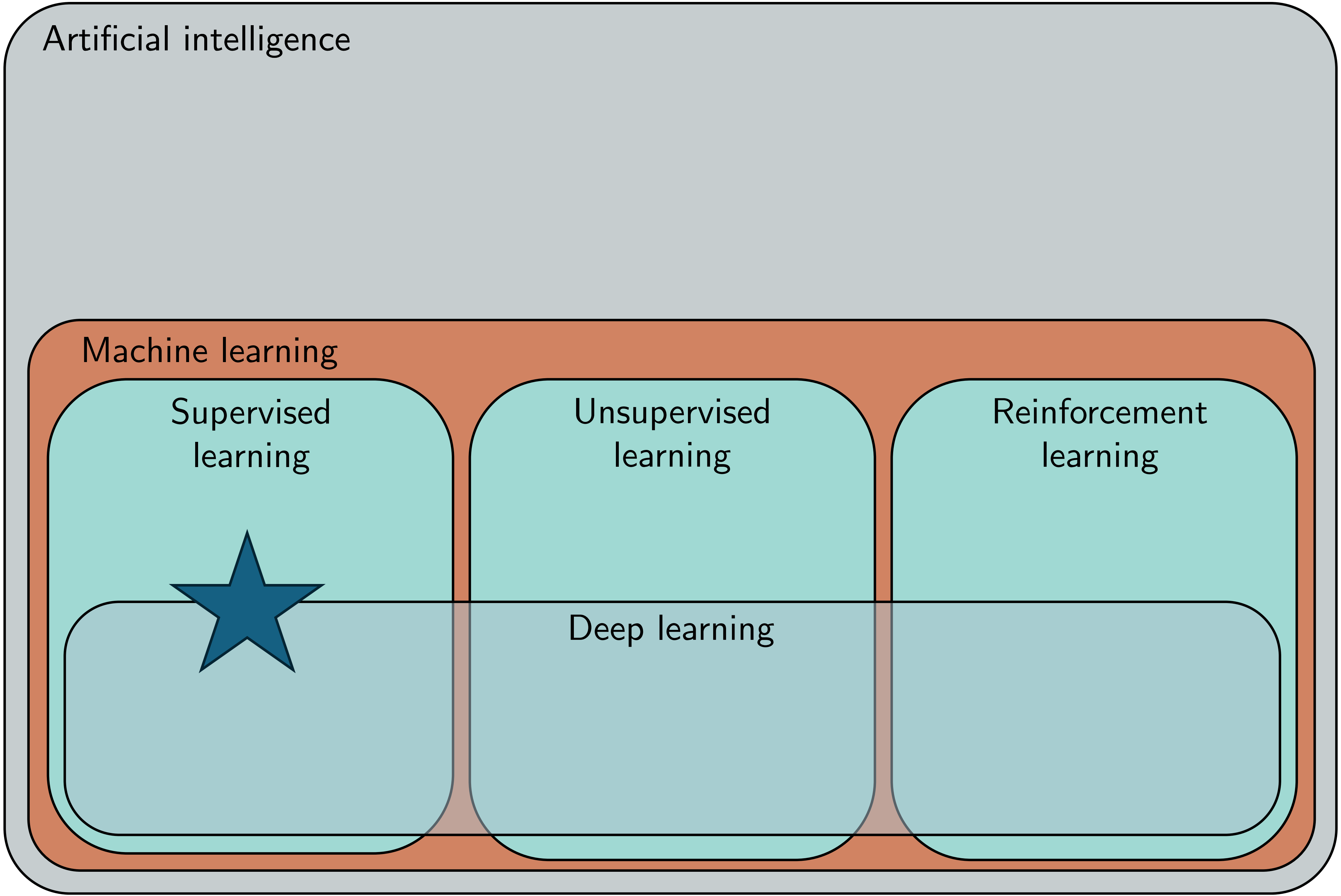
https://dl4ds.github.io/fa2025/

Supervised Learning

Original slides by Tom Gardos,
other content from Understanding Deep Learning unless otherwise

Supervised learning

- Examples
- Terminology
- Notation
  - Model
  - Loss function
  - Training
  - Testing
- 1D Linear regression example
  - Model
  - Loss function
  - Training
  - Testing



Artificial intelligence

Machine learning

Supervised learning

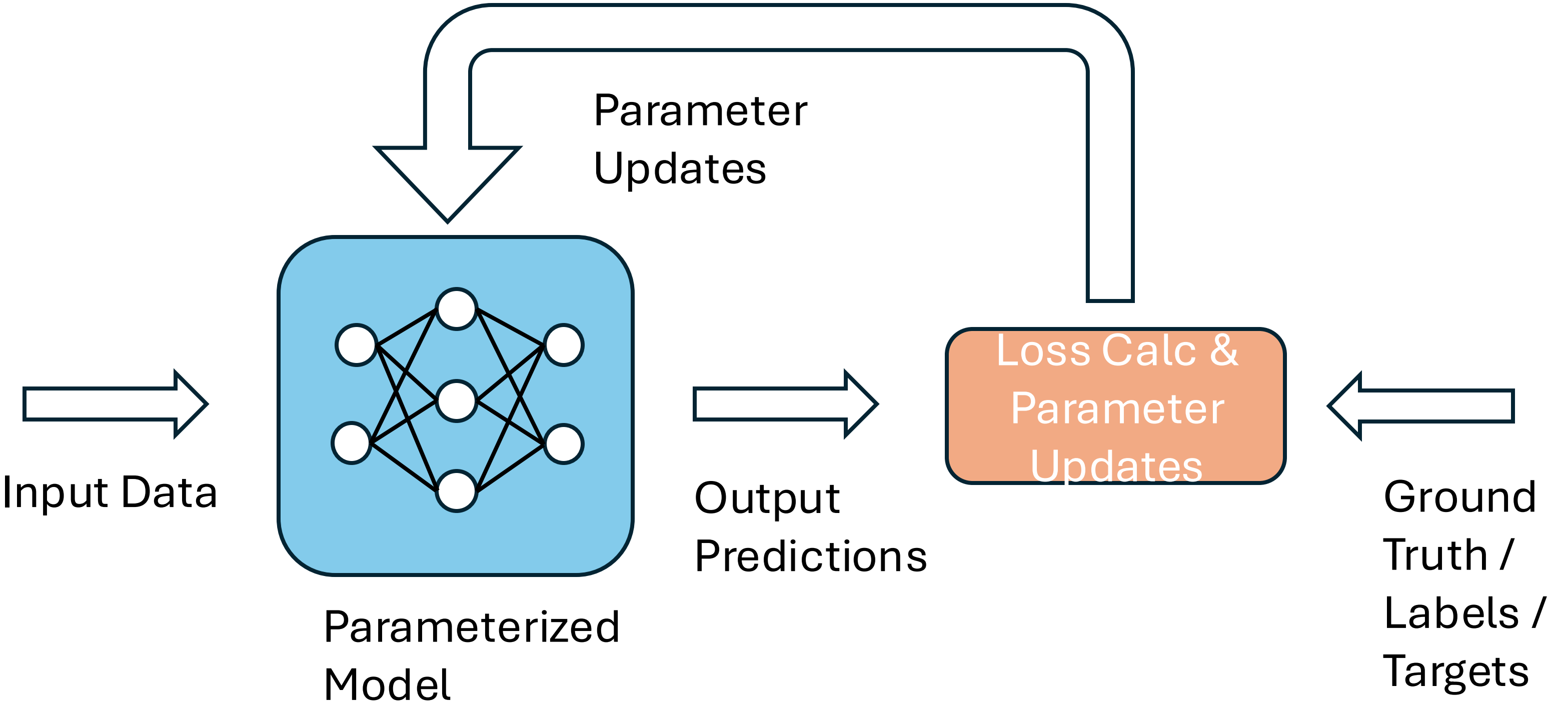
Unsupervised learning

Reinforcement learning

Deep learning

Supervised learning

- Define a mapping from input to output
- Learn this mapping from paired input/output data examples



Parameter Updates

Input Data

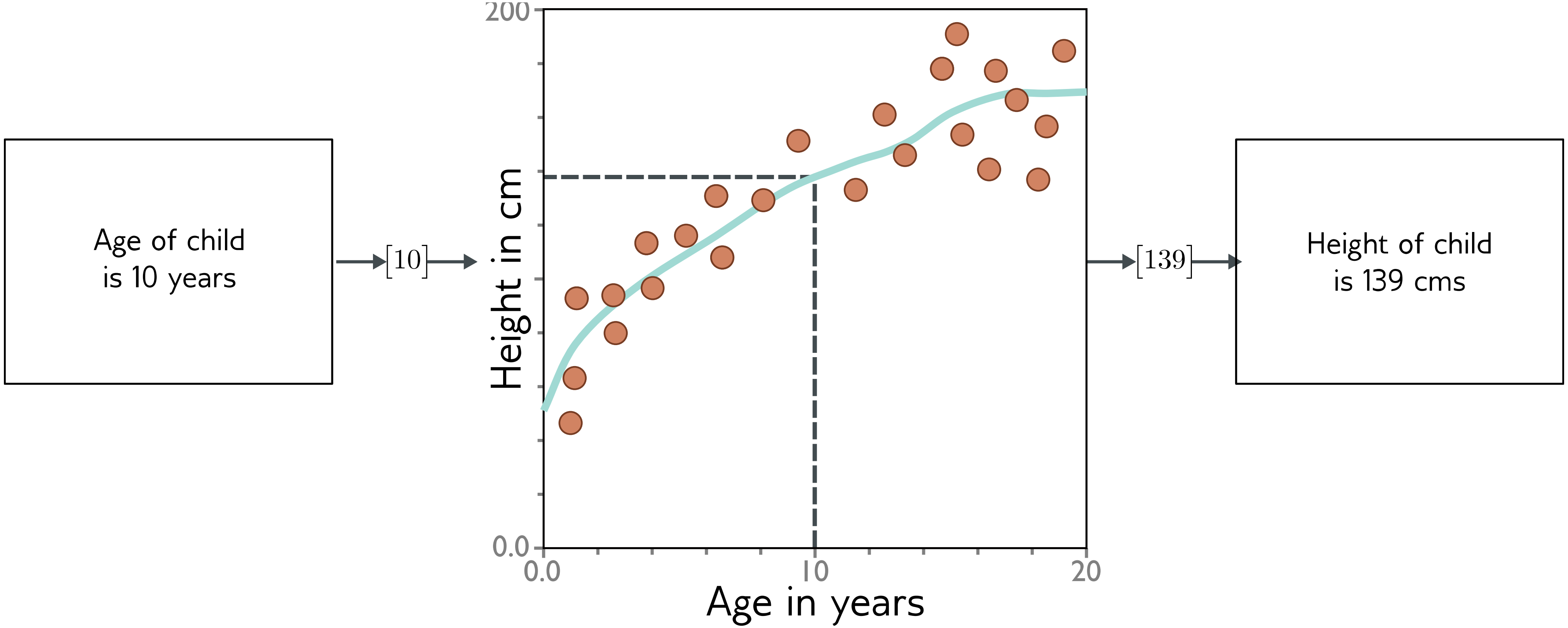
Parameterized Model

Output Predictions

Loss Calc & Parameter Updates

Ground Truth / Labels / Targets

What is a supervised learning model?

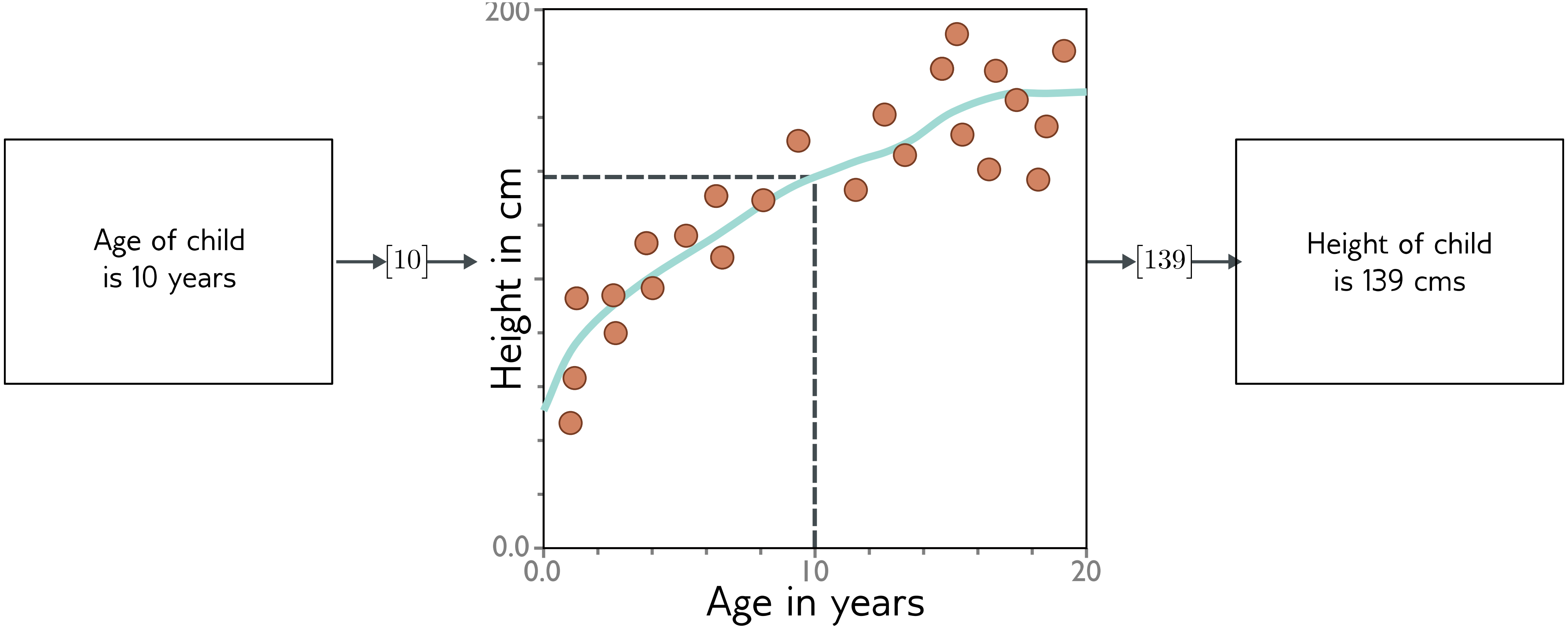


Age of child
is 10 years

Height of child
is 139 cms

- An equation relating input (age) to output (height)
- Search through family of possible equations to find one that fits training data well

What is a supervised learning model?



Age of child
is 10 years

[10]

[139]

Height of child
is 139 cms

- Deep neural networks are just a very flexible family of equations
- Fitting deep neural networks = “Deep Learning”

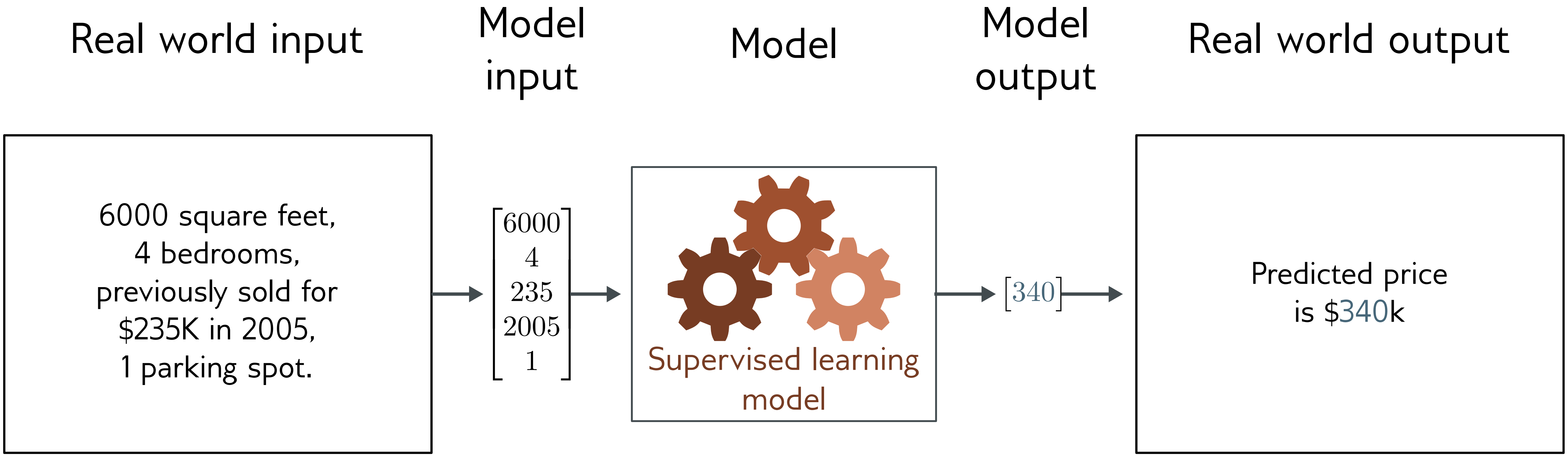
Prediction Types

- Regression
  - Prediction a continuous valued output

- Classification
  - Assigning input to one of a finite number of classes or categories
  - Two classes are a special case

Can be univariate (one output) or multivariate (more than one output)

Regression



Real world input

Model input

Model

Model output

Real world output

6000 square feet,
4 bedrooms,
previously sold for
$235K in 2005,
1 parking spot.

$$\begin{bmatrix} 6000 \\ 4 \\ 235 \\ 2005 \\ 1 \end{bmatrix}$$

Supervised learning model

$$[340]$$

Predicted price is $340k

- Univariate regression problem (one output, real value)
- Fully connected network

Graph regression

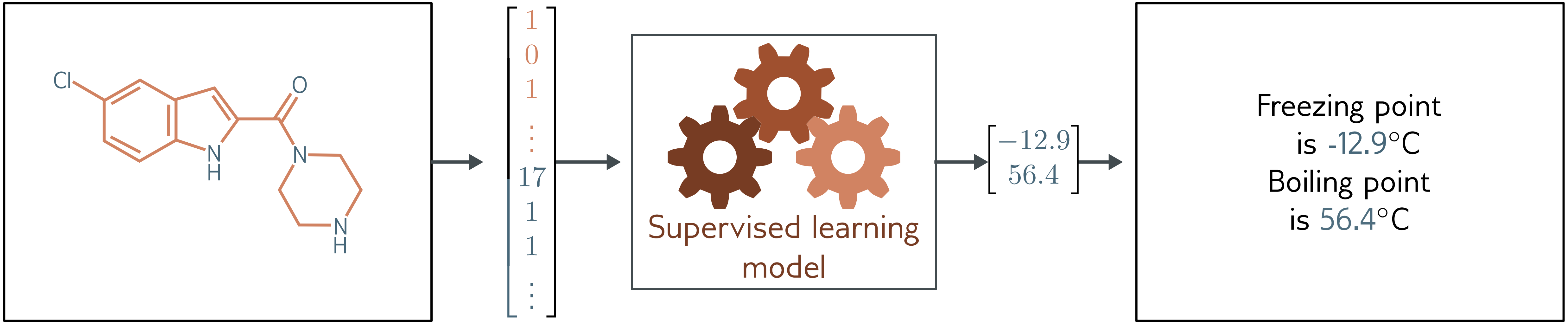
Real world input

Model input

Model

Model output

Real world output



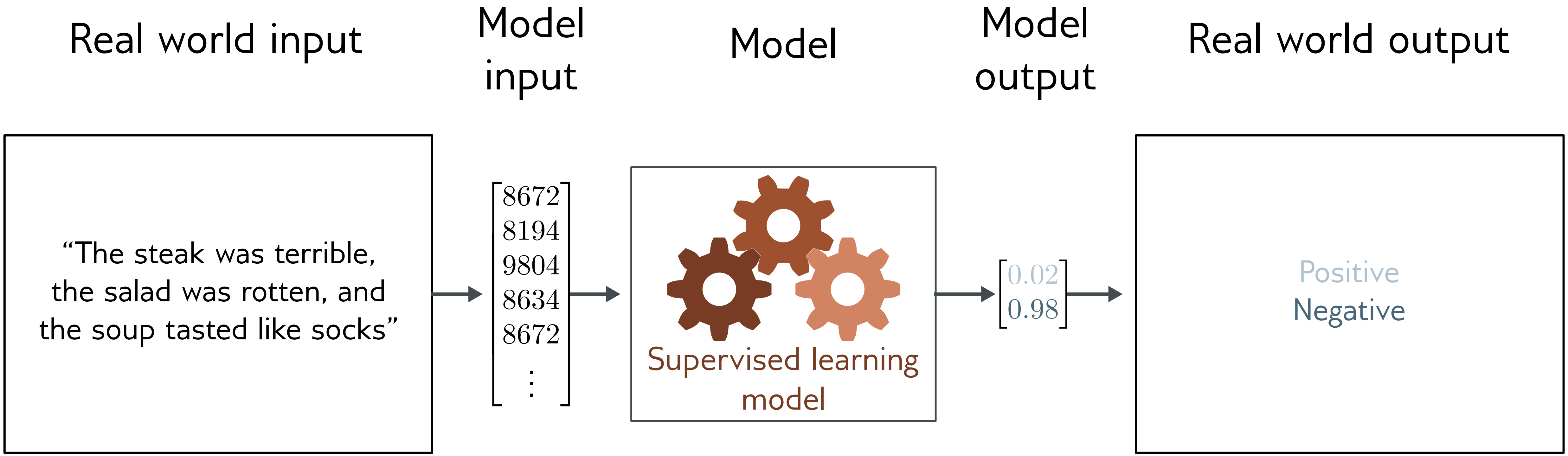
$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ \vdots \\ 17 \\ 1 \\ 1 \\ \vdots \end{bmatrix}$$

$$\begin{bmatrix} -12.9 \\ 56.4 \end{bmatrix}$$

Freezing point is -12.9°C
Boiling point is 56.4°C

- Multivariate regression problem (>1 output, real value)
- Graph neural network

Text classification



Real world input

Model input

Model

Model output

Real world output

"The steak was terrible, the salad was rotten, and the soup tasted like socks"

$$\begin{bmatrix} 8672 \\ 8194 \\ 9804 \\ 8634 \\ 8672 \\ \vdots \end{bmatrix}$$

Supervised learning model

$$\begin{bmatrix} 0.02 \\ 0.98 \end{bmatrix}$$

Positive
Negative

- Binary classification problem (two discrete classes)
- Transformer network

Music genre classification

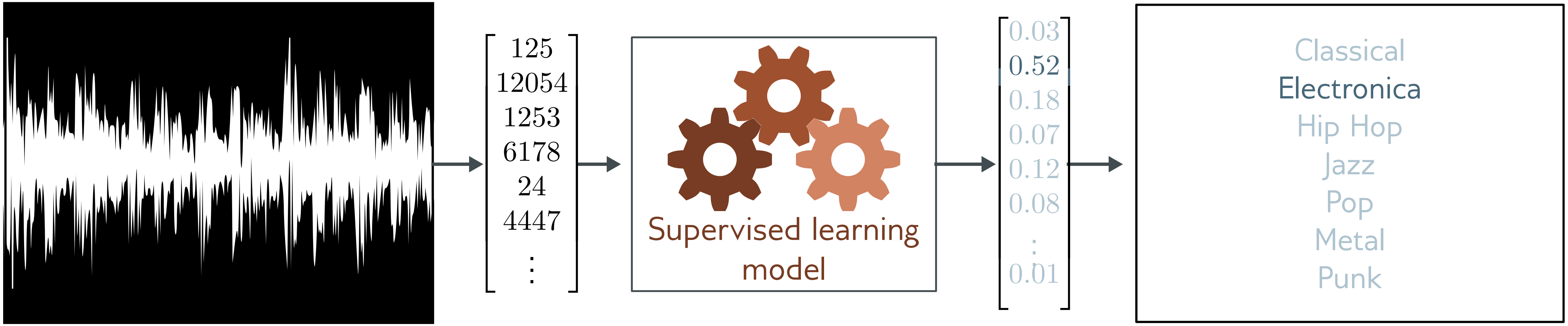
Real world input

Model input

Model

Model output

Real world output



$$\begin{bmatrix} 125 \\ 12054 \\ 1253 \\ 6178 \\ 24 \\ 4447 \\ \vdots \end{bmatrix}$$

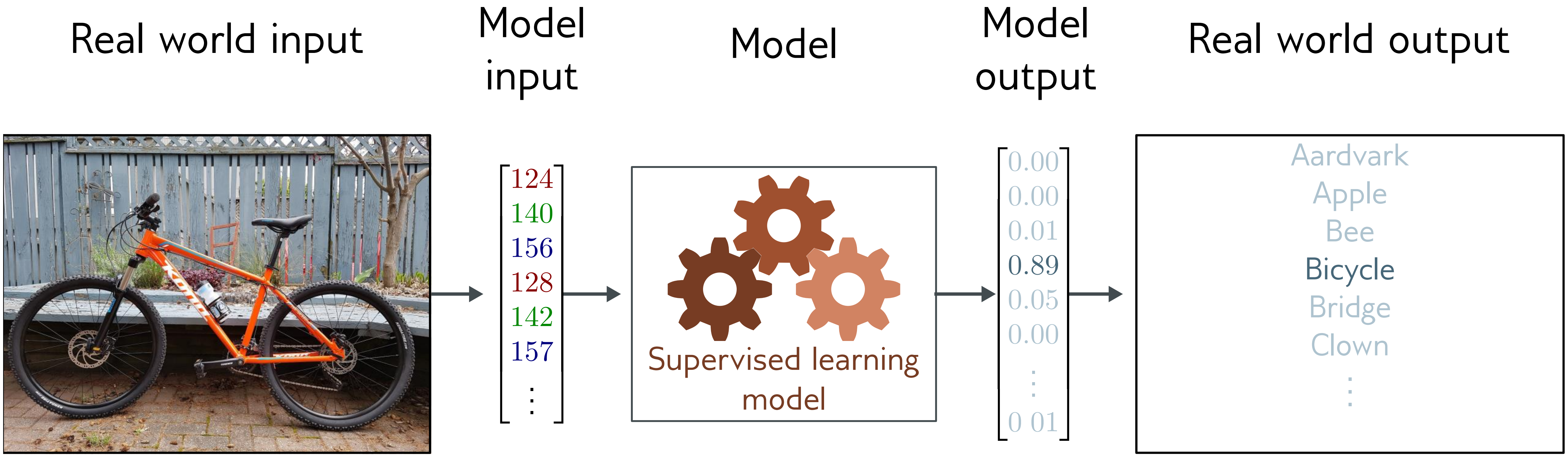
Supervised learning model

$$\begin{bmatrix} 0.03 \\ 0.52 \\ 0.18 \\ 0.07 \\ 0.12 \\ 0.08 \\ \vdots \\ 0.01 \end{bmatrix}$$

Classical
Electronica
Hip Hop
Jazz
Pop
Metal
Punk

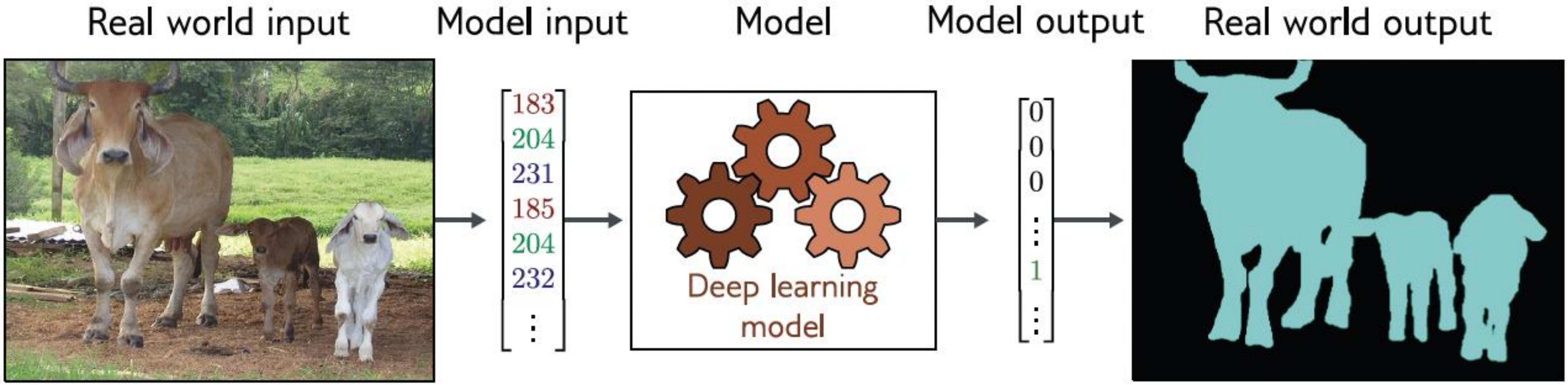
- Multiclass classification problem (discrete classes, >2 possible values)
- Recurrent neural network (RNN)

Image classification



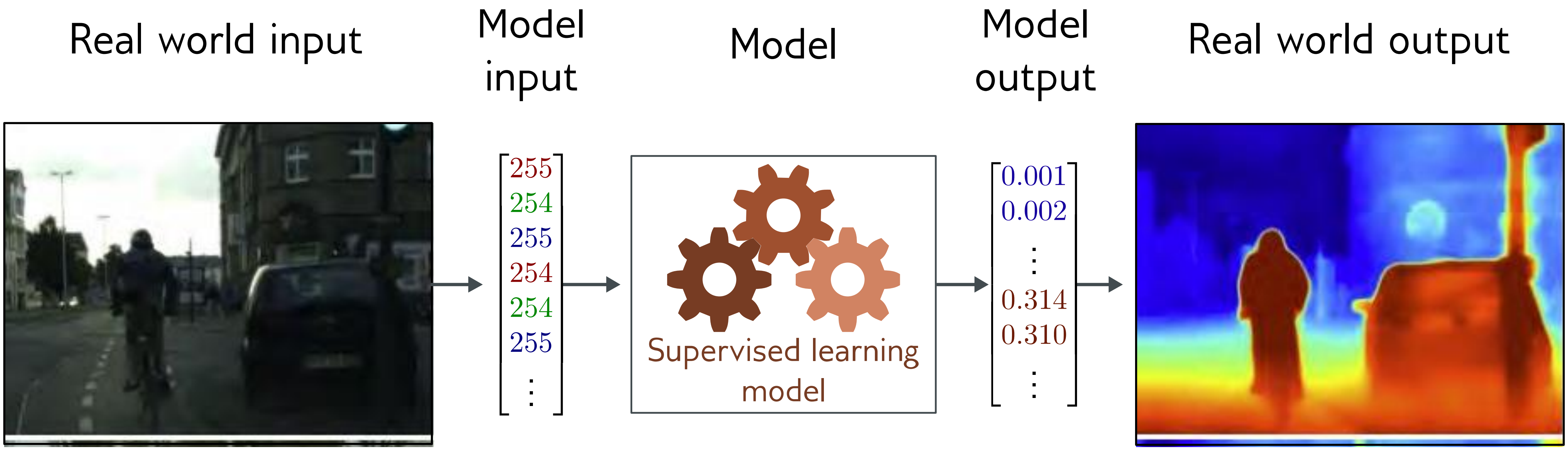
- Multiclass classification problem (discrete classes, >2 possible classes)
- Convolutional network

Image segmentation



- Multivariate binary classification problem (many outputs, two discrete classes)
- Convolutional encoder-decoder network

Depth estimation



Real world input

Model input

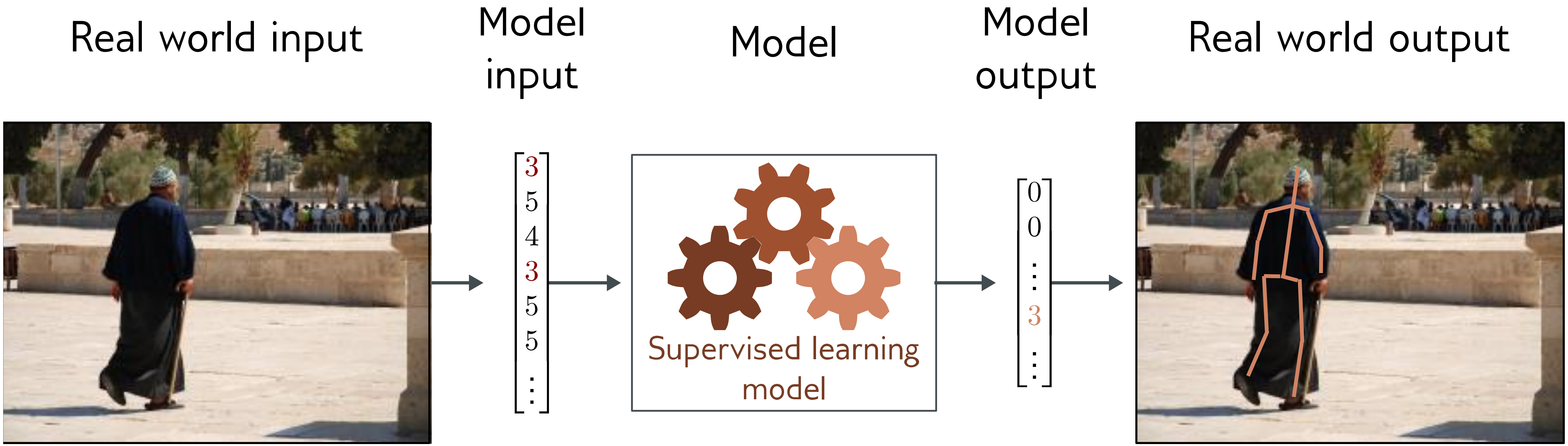
Model

Model output

Real world output

- Multivariate regression problem (many outputs, continuous)
- Convolutional encoder-decoder network

Pose estimation



Real world input

Model input

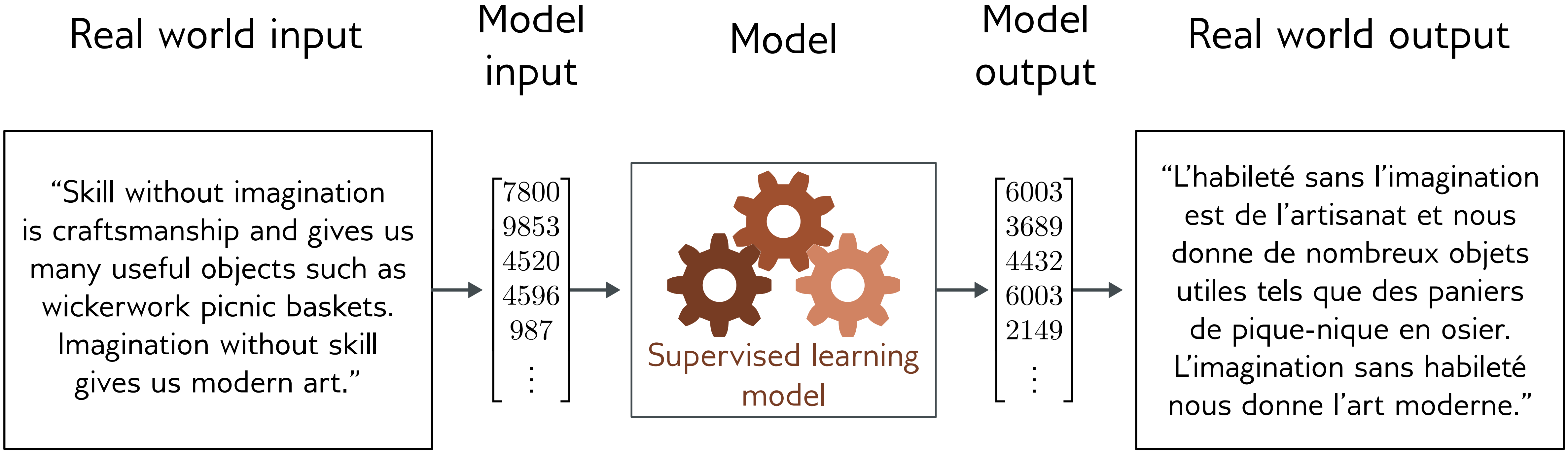
Model

Model output

Real world output

- Multivariate regression problem (many outputs, continuous)
- Convolutional encoder-decoder network

Translation



Real world input

Model input

Model

Model output

Real world output

"Skill without imagination is craftsmanship and gives us many useful objects such as wickerwork picnic baskets. Imagination without skill gives us modern art."

$$\begin{bmatrix} 7800 \\ 9853 \\ 4520 \\ 4596 \\ 987 \\ \vdots \end{bmatrix}$$

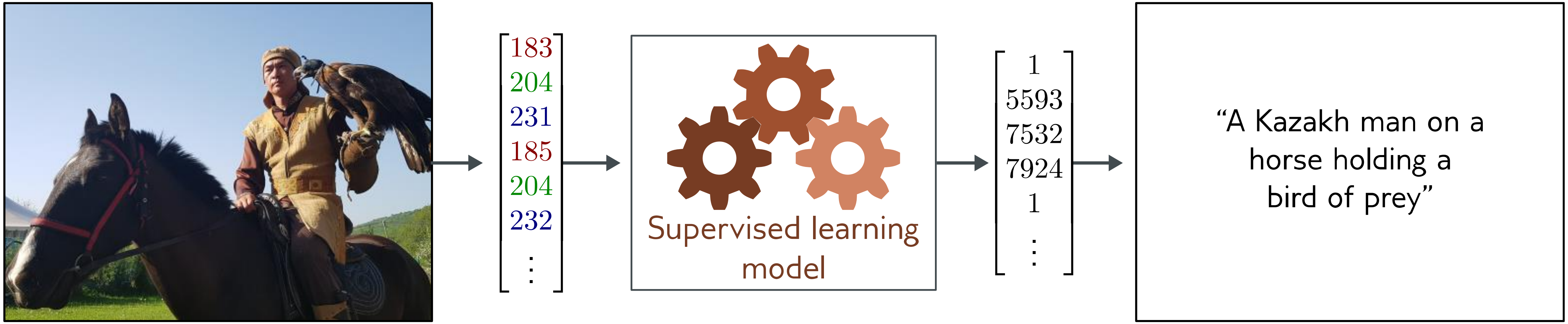
Supervised learning model

$$\begin{bmatrix} 6003 \\ 3689 \\ 4432 \\ 6003 \\ 2149 \\ \vdots \end{bmatrix}$$

"L'habileté sans l'imagination est de l'artisanat et nous donne de nombreux objets utiles tels que des paniers de pique-nique en osier. L'imagination sans habileté nous donne l'art moderne."

- Encoder-Decoder Transformer Networks

Image captioning



Real world input | Model input | Model | Model output | Real world output

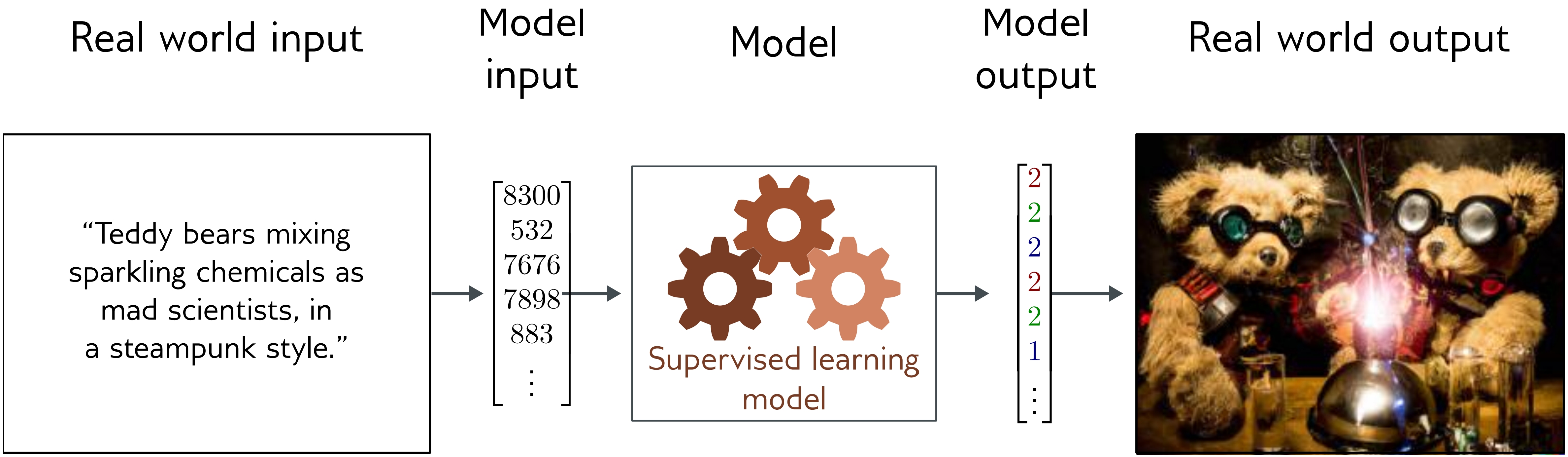
$$\begin{bmatrix}183\\204\\231\\185\\204\\232\\\vdots\end{bmatrix}$$

$$\begin{bmatrix}1\\5593\\7532\\7924\\1\\\vdots\end{bmatrix}$$

"A Kazakh man on a horse holding a bird of prey"

- E.g. CNN-RNN, LSTM, Transformers

Image generation from text



Real world input

Model input

Model

Model output

Real world output

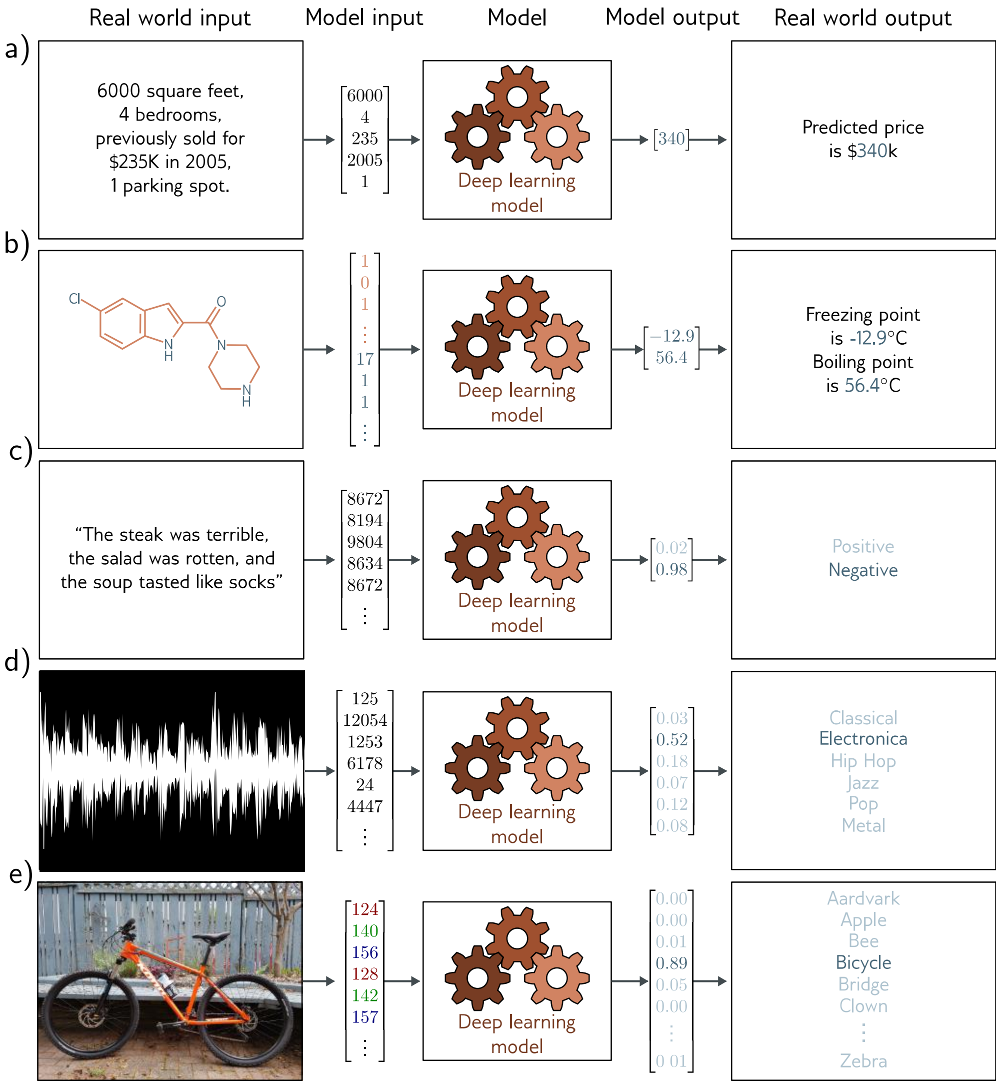
"Teddy bears mixing sparkling chemicals as mad scientists, in a steampunk style."

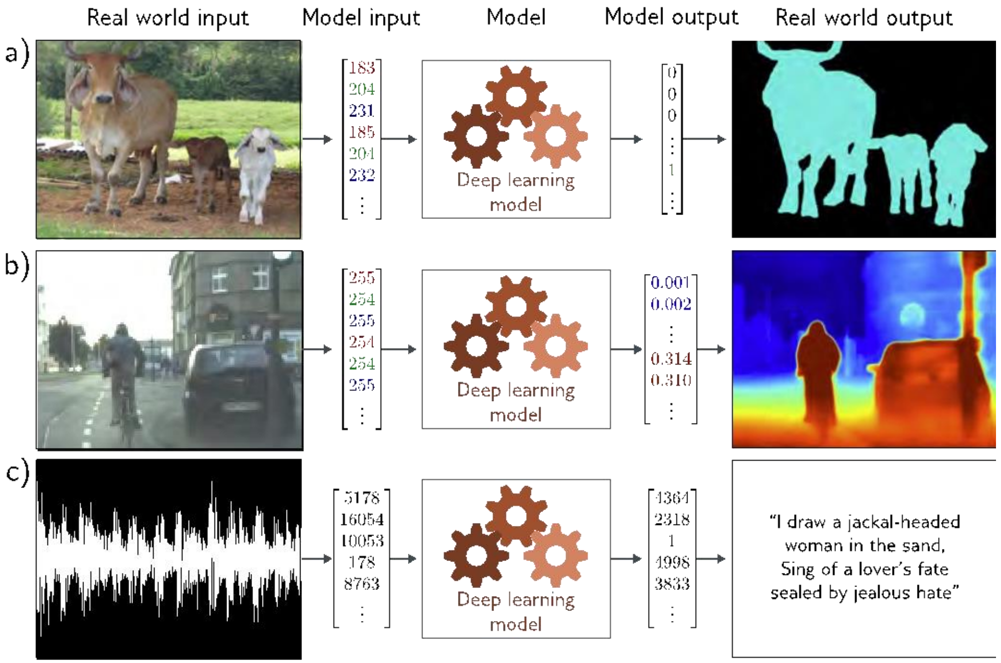
$$\begin{bmatrix}8300\\532\\7676\\7898\\883\\\vdots\end{bmatrix}$$

Supervised learning model

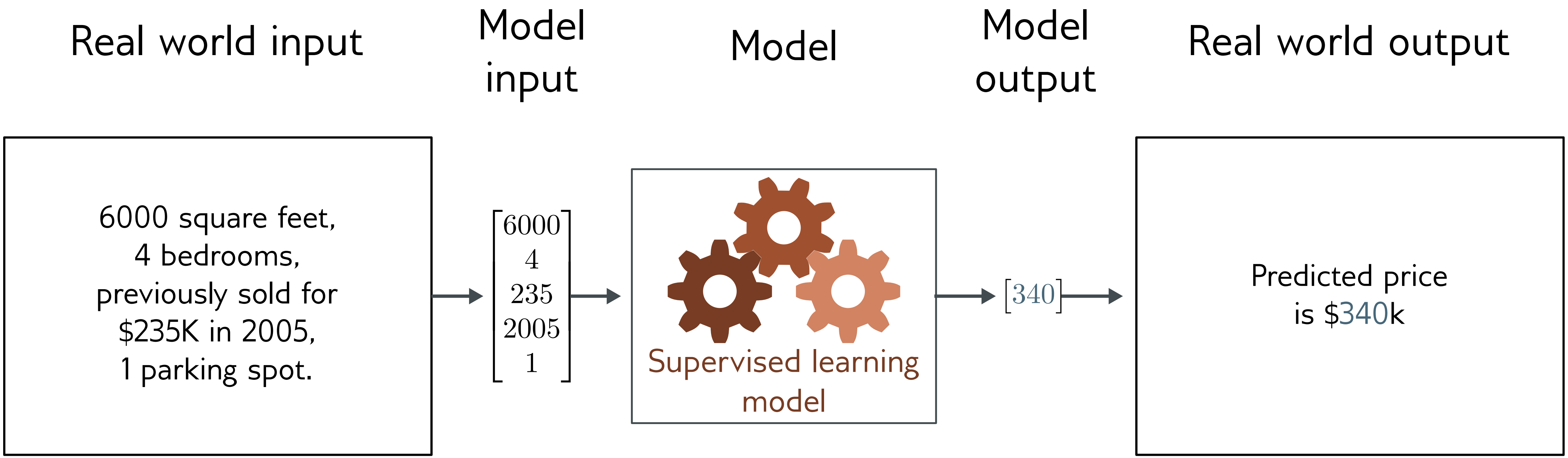
$$\begin{bmatrix}2\\2\\2\\2\\2\\1\\\vdots\end{bmatrix}$$

Supervised Learning Classification and Regression Applications





Regression



Real world input

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Model input

$$\begin{bmatrix} 6000 \\ 4 \\ 235 \\ 2005 \\ 1 \end{bmatrix}$$

Model

Supervised learning model

Model output

$$[340]$$

Real world output

Predicted price
is $340k

- Univariate regression problem (one output, real value)

Any Questions?

Supervised learning

- Examples
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Supervised learning terminology

- Supervised learning model = mapping from one or more inputs to one or more outputs
- Model is a family of equations → "inductive bias"
- Computing the outputs from the inputs → inference
- Model also includes parameters
- Parameters affect outcome of equation
- Training a model = finding parameters that predict outputs "well" from inputs for training and evaluation datasets of input/output pairs

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Also Appendix A of the book.

Notation:

- Input:

$$\mathbf{x}$$

- Output:

$$\mathbf{y}$$

- Model:

$$\mathbf{y} = \mathbf{f}[\mathbf{x}]$$

Variables always Roman letters

Normal lower case = scalar
Bold lower case = vector
Capital Bold = matrix

Functions always square brackets

Normal lower case = returns scalar
Bold lower case = returns vector
Capital Bold = returns matrix

Notation example:

- Input:

$$\mathbf{x} = \begin{bmatrix} \text{age} \\ \text{mileage} \end{bmatrix}$$

Vector: Structured or tabular data

- Output:

$$y = \begin{bmatrix} \text{price} \end{bmatrix}$$

Scalar output

- Model:

$$y = \mathrm{f}[\mathbf{x}]$$

Scalar output function (with vector input)

Model

- Parameters:

$$\phi$$

Parameters always Greek letters

- Model :

$$\mathbf{y} = \mathbf{f}[\mathbf{x}, \phi]$$

Data Set and Loss function

- Training dataset of I pairs of input/output examples:

$$\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{I}$$

Data Set and Loss function

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- Loss function or cost function measures how bad model is:

$$L\Bigl[\phi, \underbrace{\mathrm{f}[\mathbf{x}, \phi]}_{\text{model}}, \underbrace{\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{I}}_{\text{train data}}\Bigr]$$

Data Set and Loss function

- Training dataset of I pairs of input/output examples:

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$$L\Big[\phi, \underbrace{\mathrm{f}[\mathbf{x}, \phi]}_{\text{model}}, \underbrace{\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{I}}_{\text{train data}}\Big]$$

or for short:

$$L\left[\phi\right] \longleftarrow$$

Returns a scalar that is smaller when model maps inputs to outputs better

Training

- Loss function:

$$L\left[\phi\right]$$

Returns a scalar that is smaller when model maps inputs to outputs better

- Find the parameters that minimize the loss:

$$\hat{\phi} = \underset{\phi}{\operatorname{argmin}}\Big[\mathrm{L}\left[\phi\right]\Big]$$

Any Questions?

Supervised learning

- Examples
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- 1D Linear regression example
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  - Testing

Example: 1D Linear regression model

- Model:

$$\begin{aligned} y &= \mathrm{f}[x, \boldsymbol{\phi}] \\ &= \phi_0 + \phi_1 x \end{aligned}$$

- Parameters

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_0 \\ \phi_1 \end{bmatrix} \quad \begin{matrix} \longleftarrow \text{y-offset} \\ \longleftarrow \text{slope} \end{matrix}$$

Example: 1D Linear regression model

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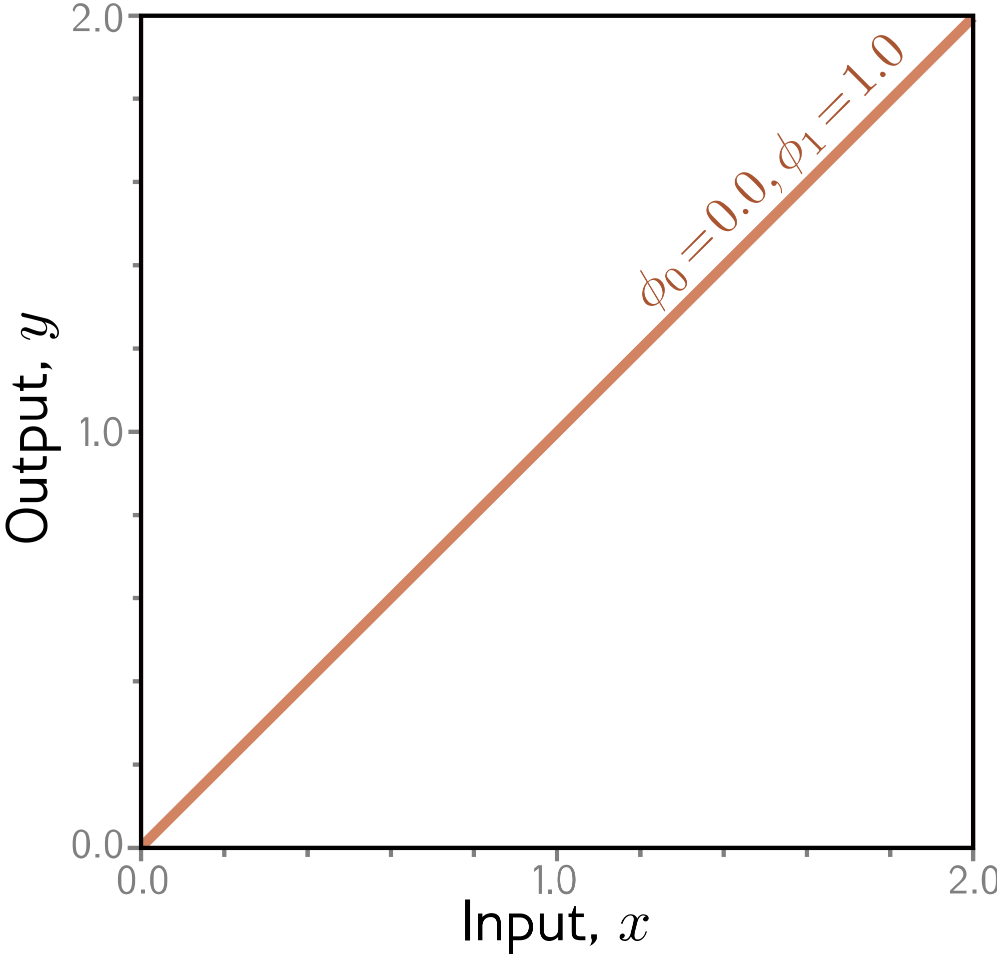
← y-offset

← slope



Example: 1D Linear regression model

- Model:

$$
\begin{aligned}
y &= \mathrm{f}[x, \boldsymbol{\phi}] \\
&= \phi_0 + \phi_1 x
\end{aligned}
$$

- Parameters

$$
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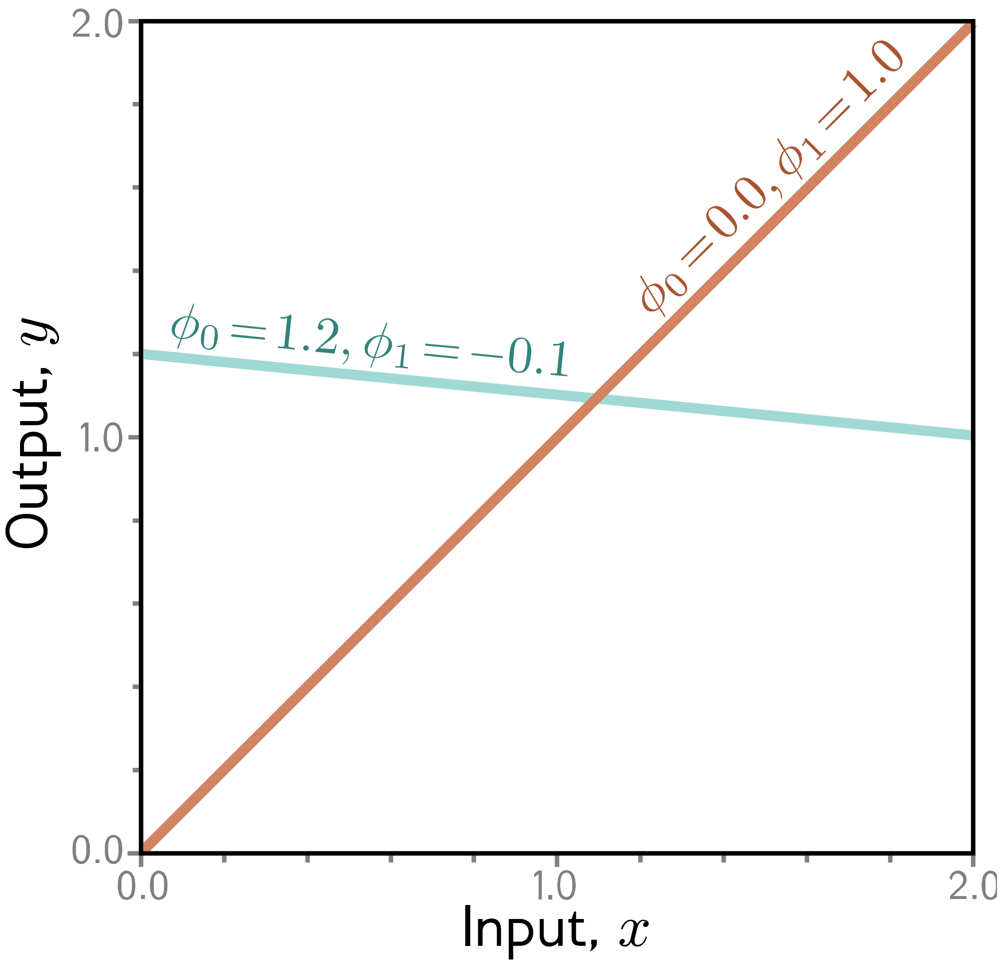
$$



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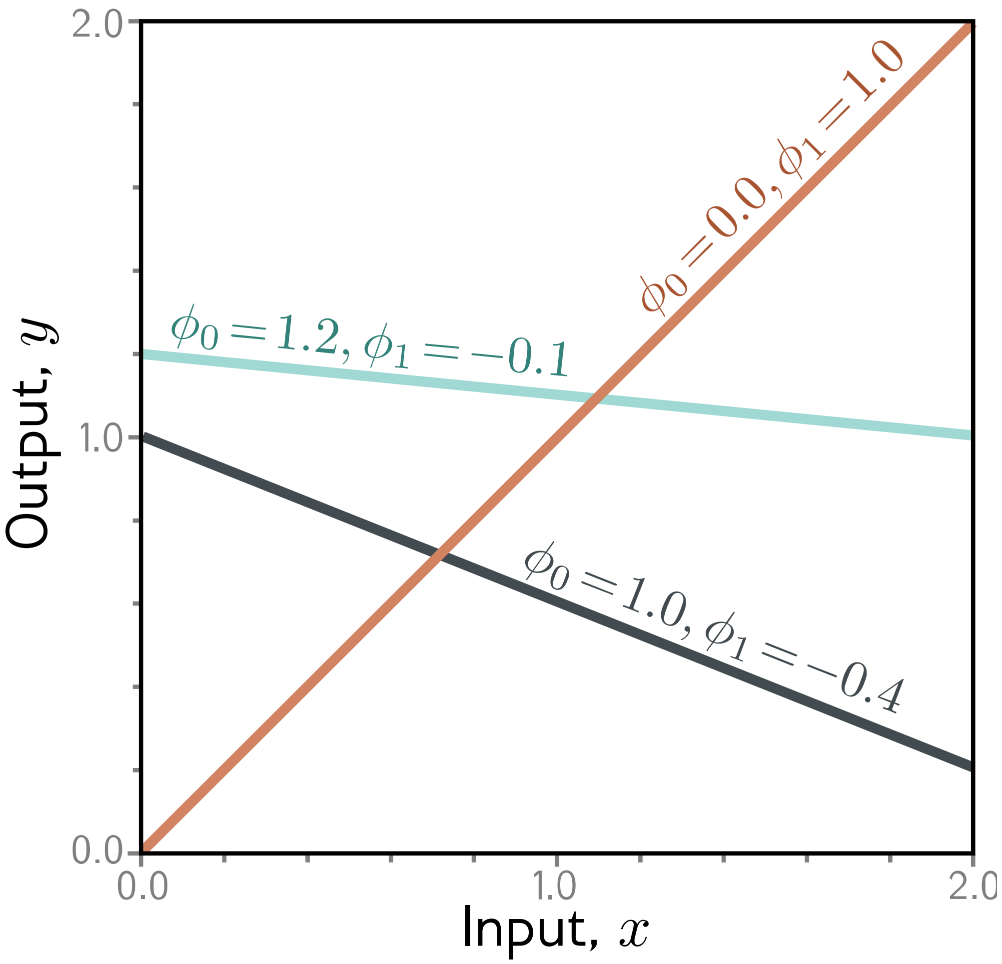
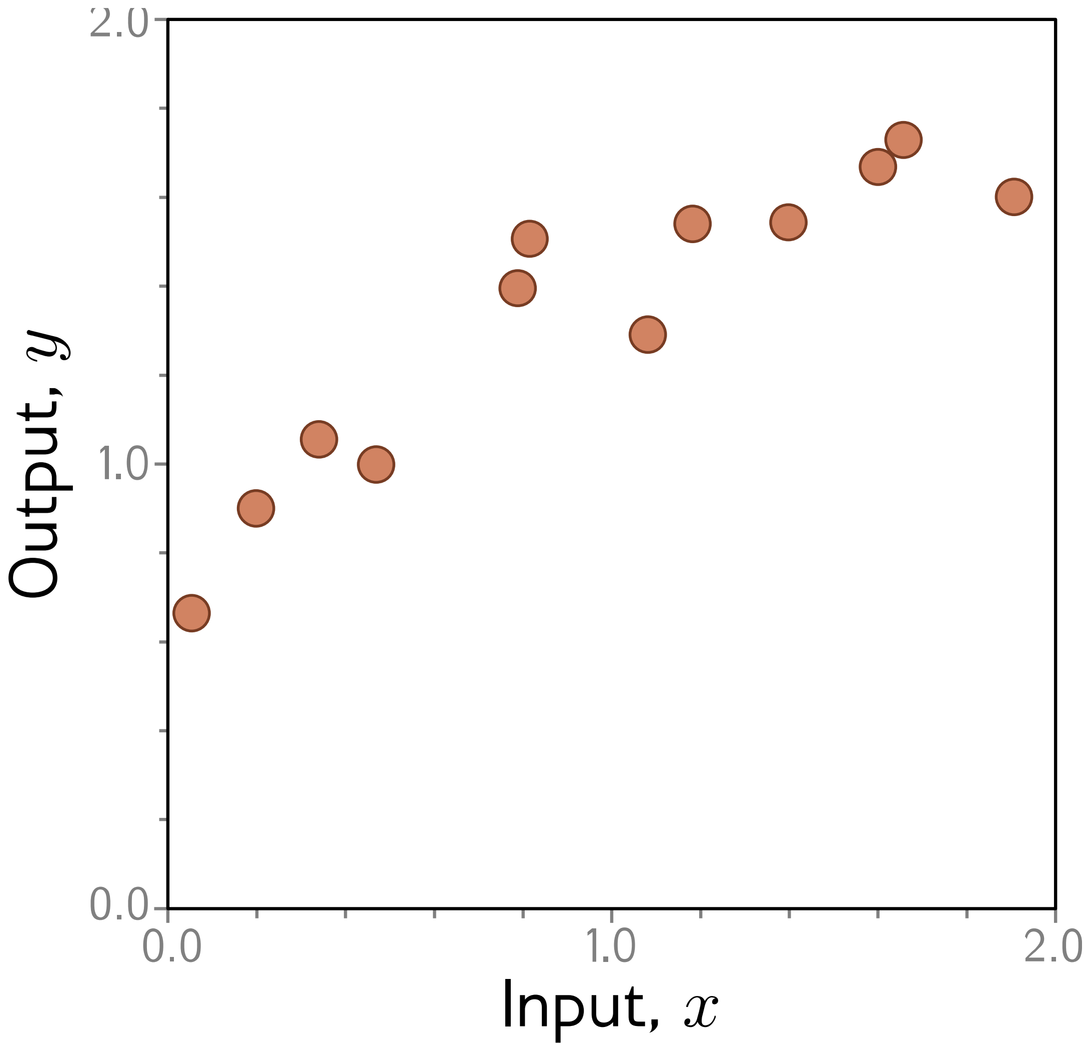


Interactive Figure 2.1

Example: 1D Linear regression training data



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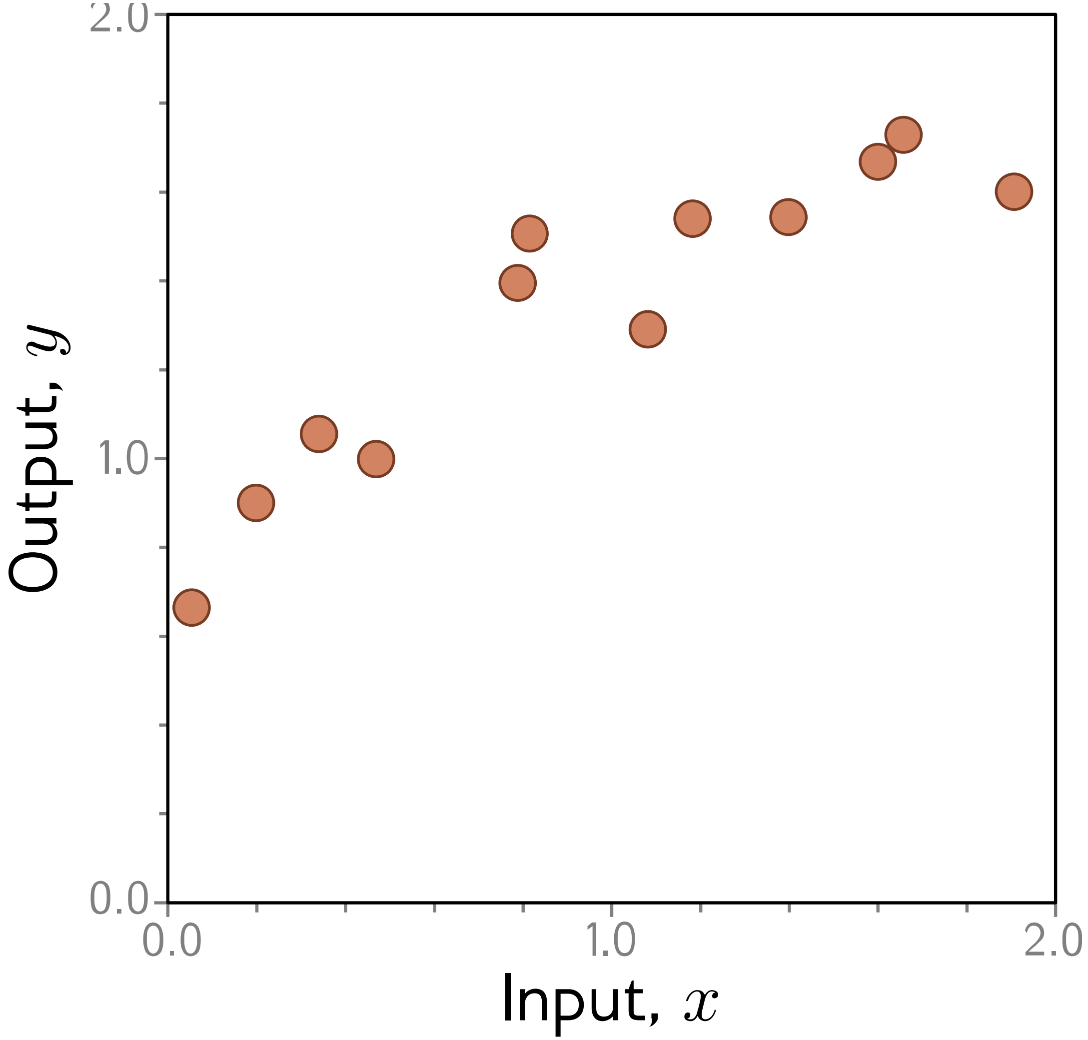


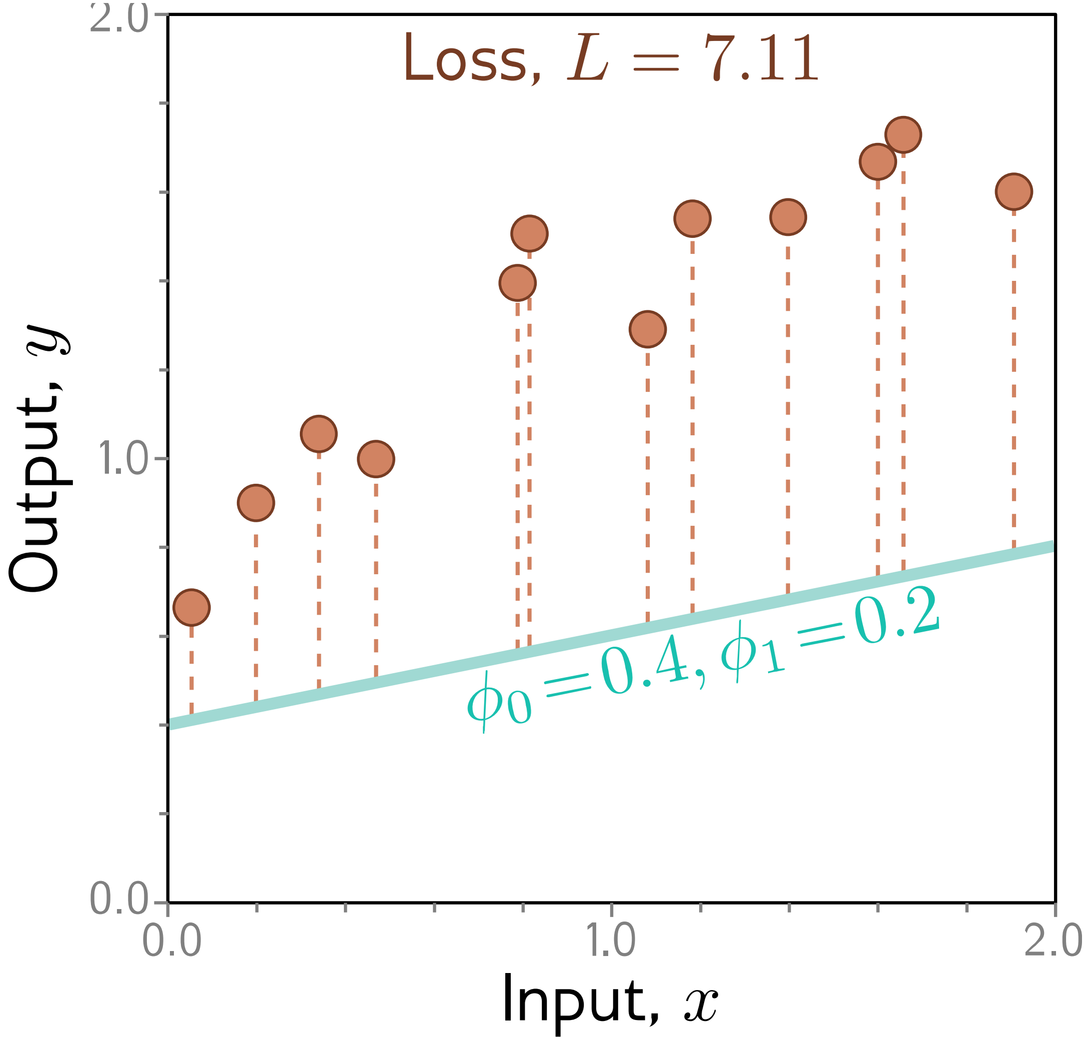
Loss function:

$$L[\boldsymbol{\phi}] = \sum_{i=1}^{I} (\mathrm{f}[x_i, \boldsymbol{\phi}] - y_i)^2$$

$$= \sum_{i=1}^{I} (\phi_0 + \phi_1 x_i - y_i)^2$$

"Least squares loss function"

Example: 1D Linear regression loss function



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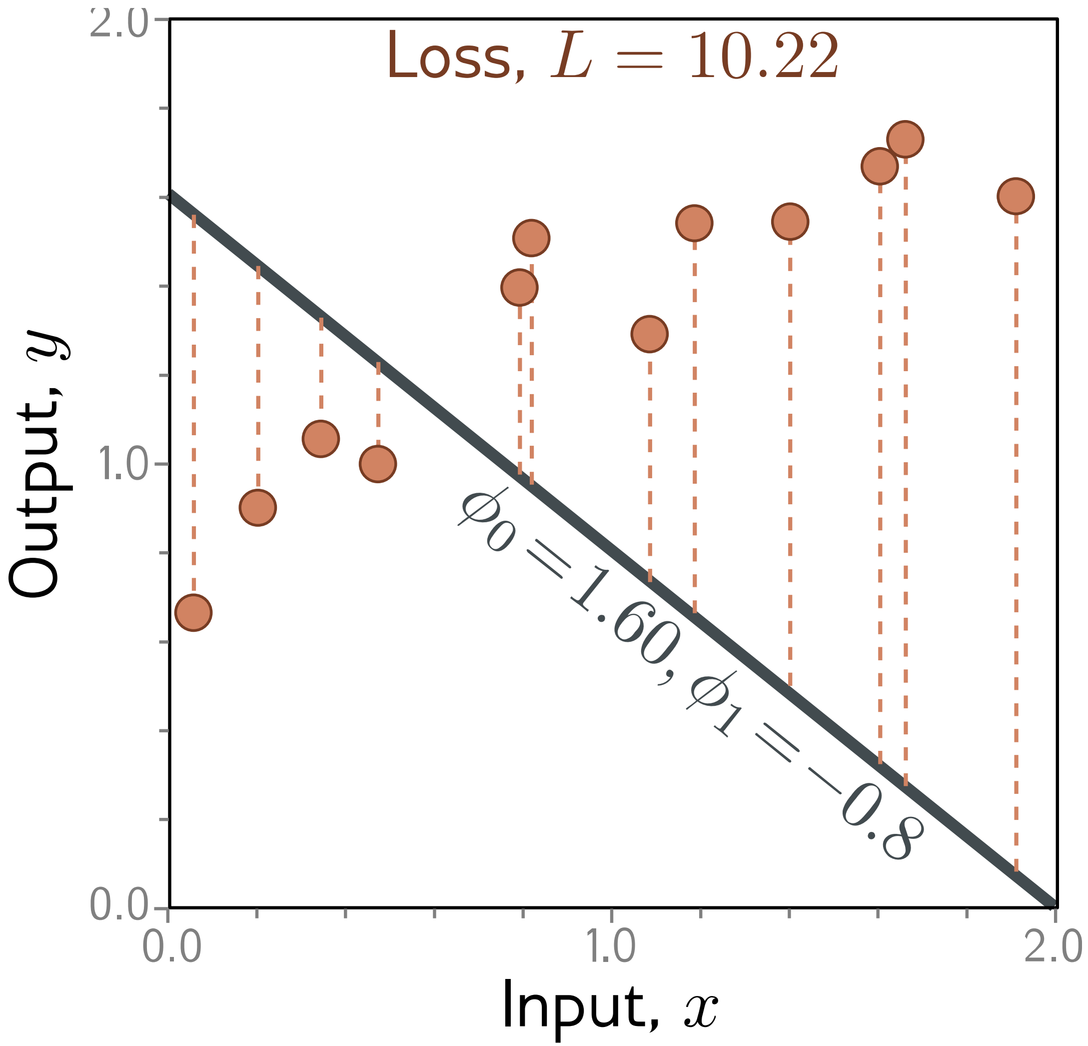

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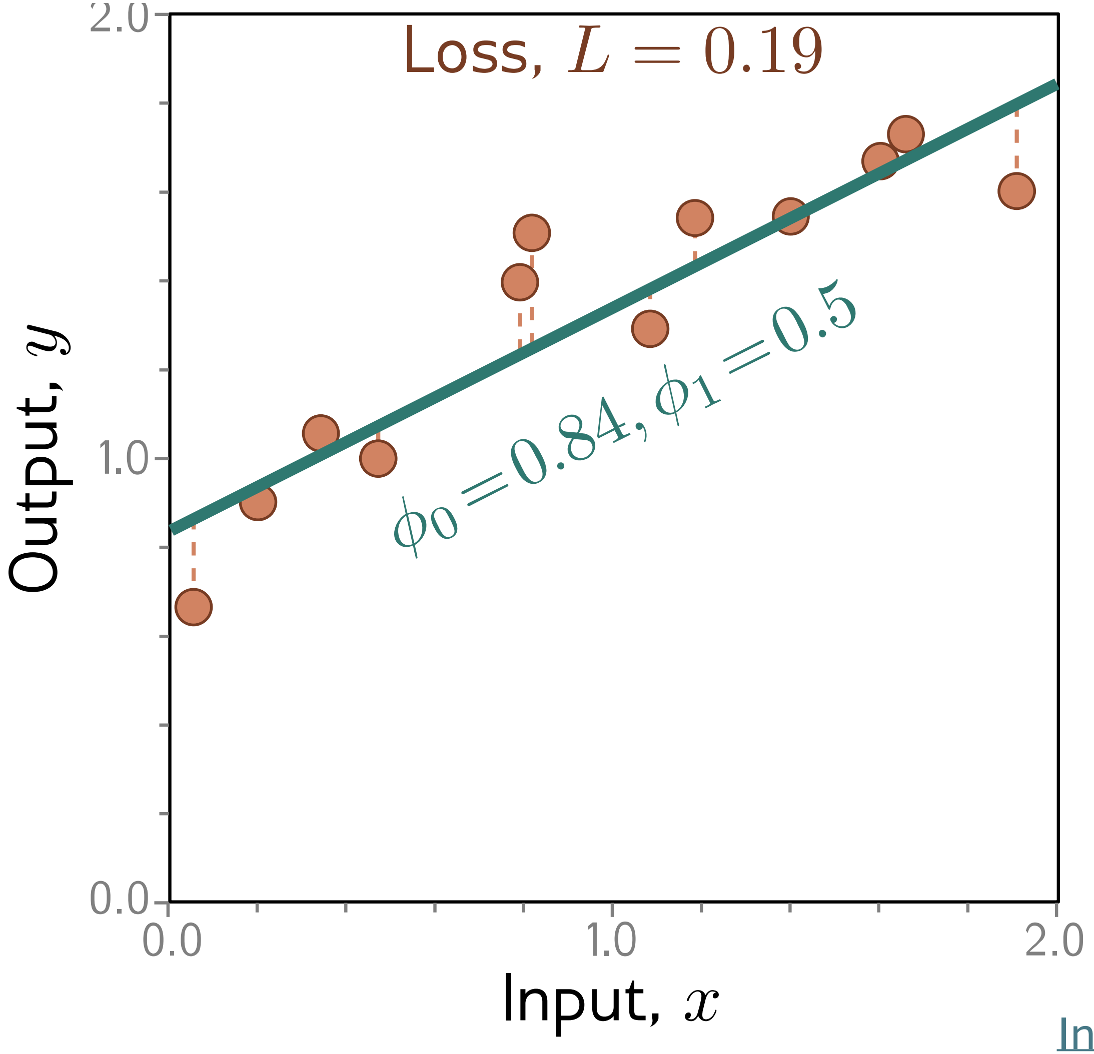

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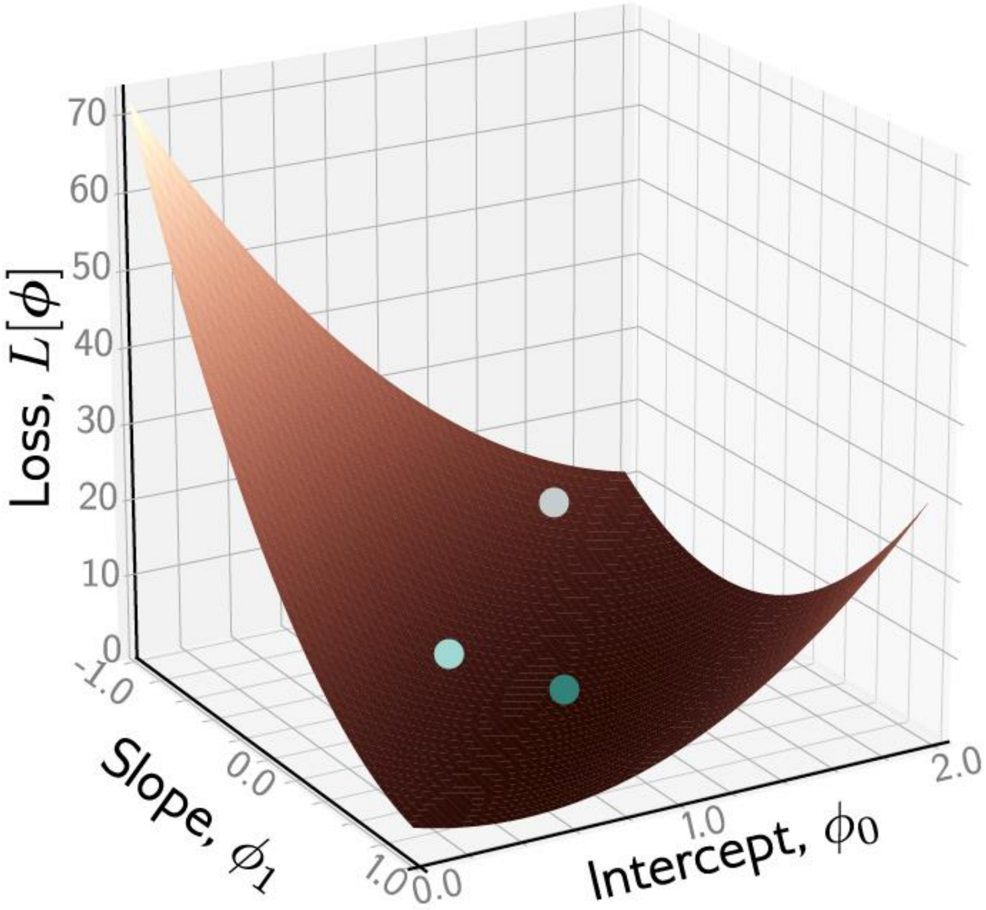
"Least squares loss function"

Interactive Figure 2.2

Example: 1D Linear regression loss function



Loss function:

$$L[\phi] = \sum_{i=1}^{I} (\mathrm{f}[x_i, \phi] - y_i)^2$$

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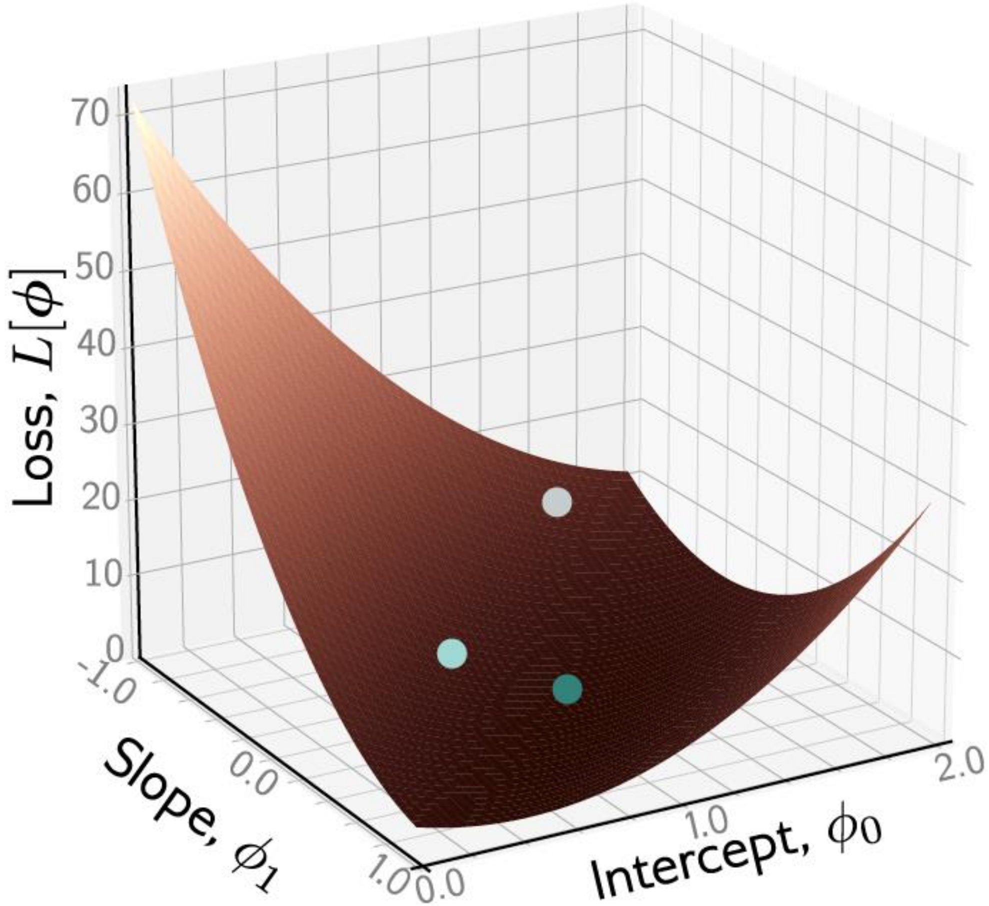
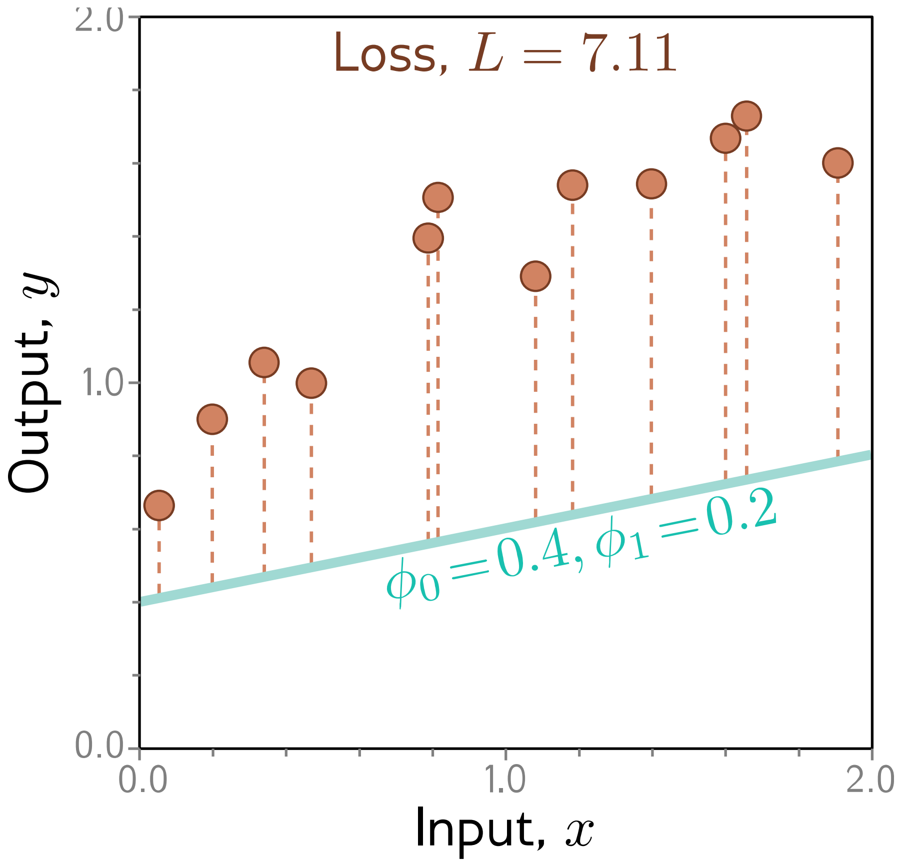
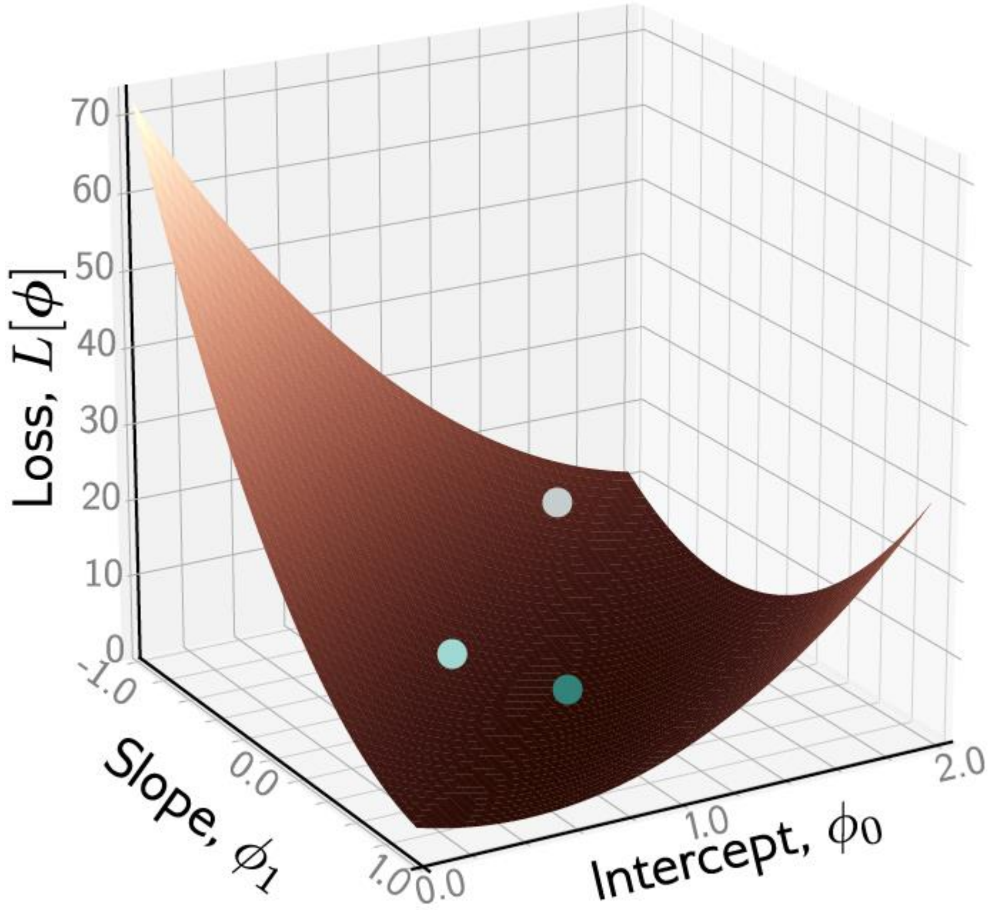
"Least squares loss function"

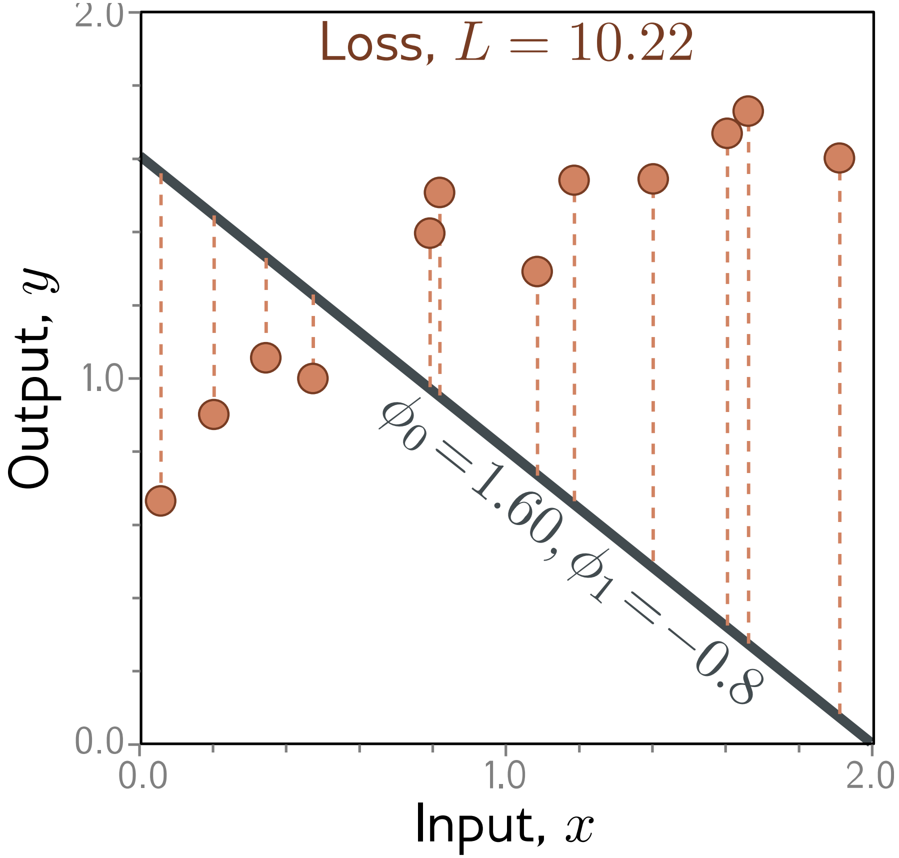
Example: 1D Linear regression loss function





Example: 1D Linear regression loss function





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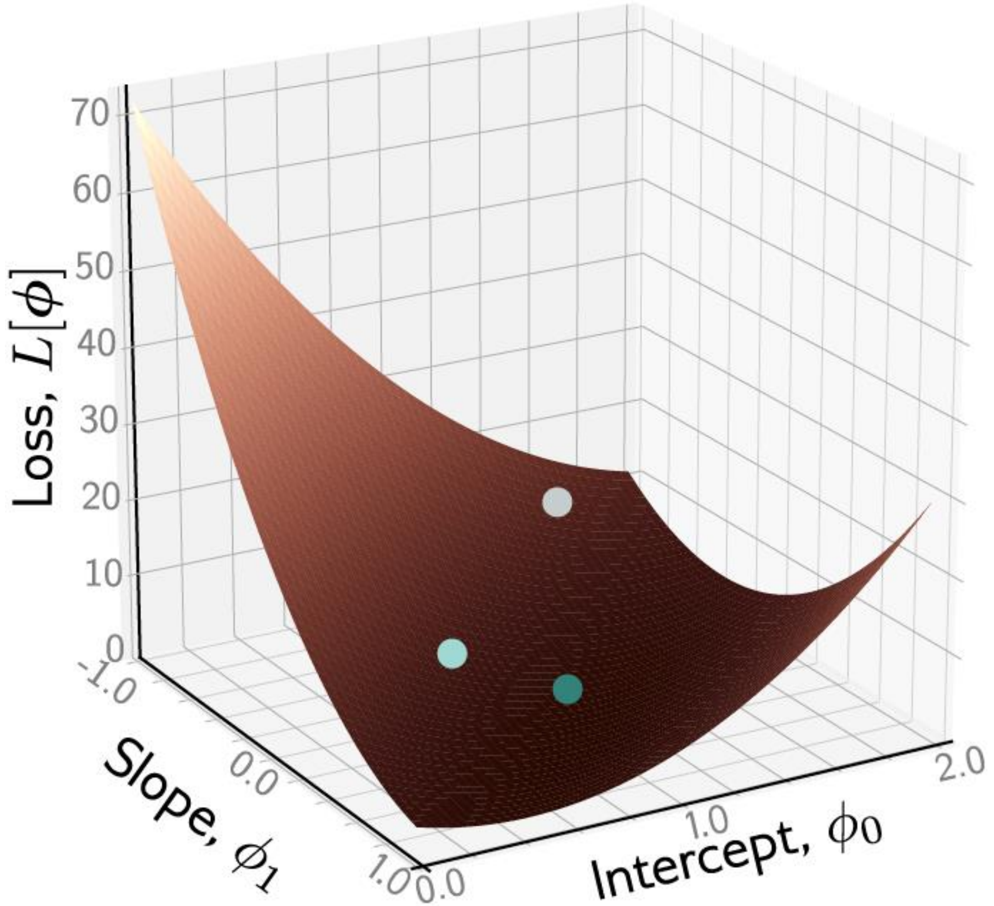

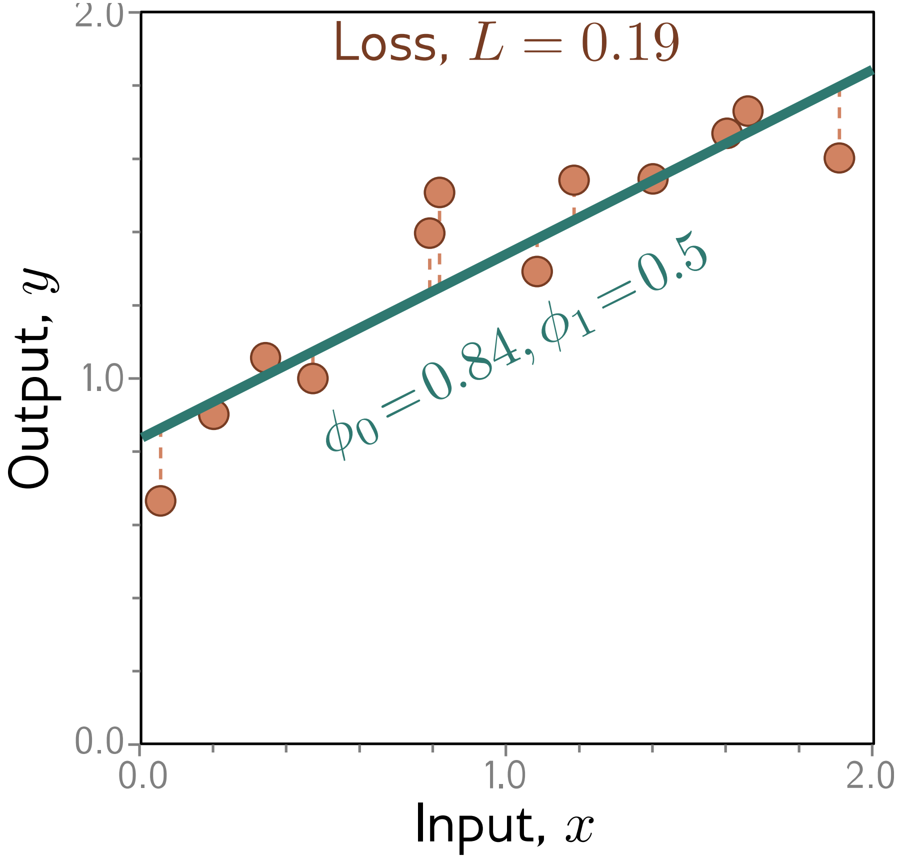

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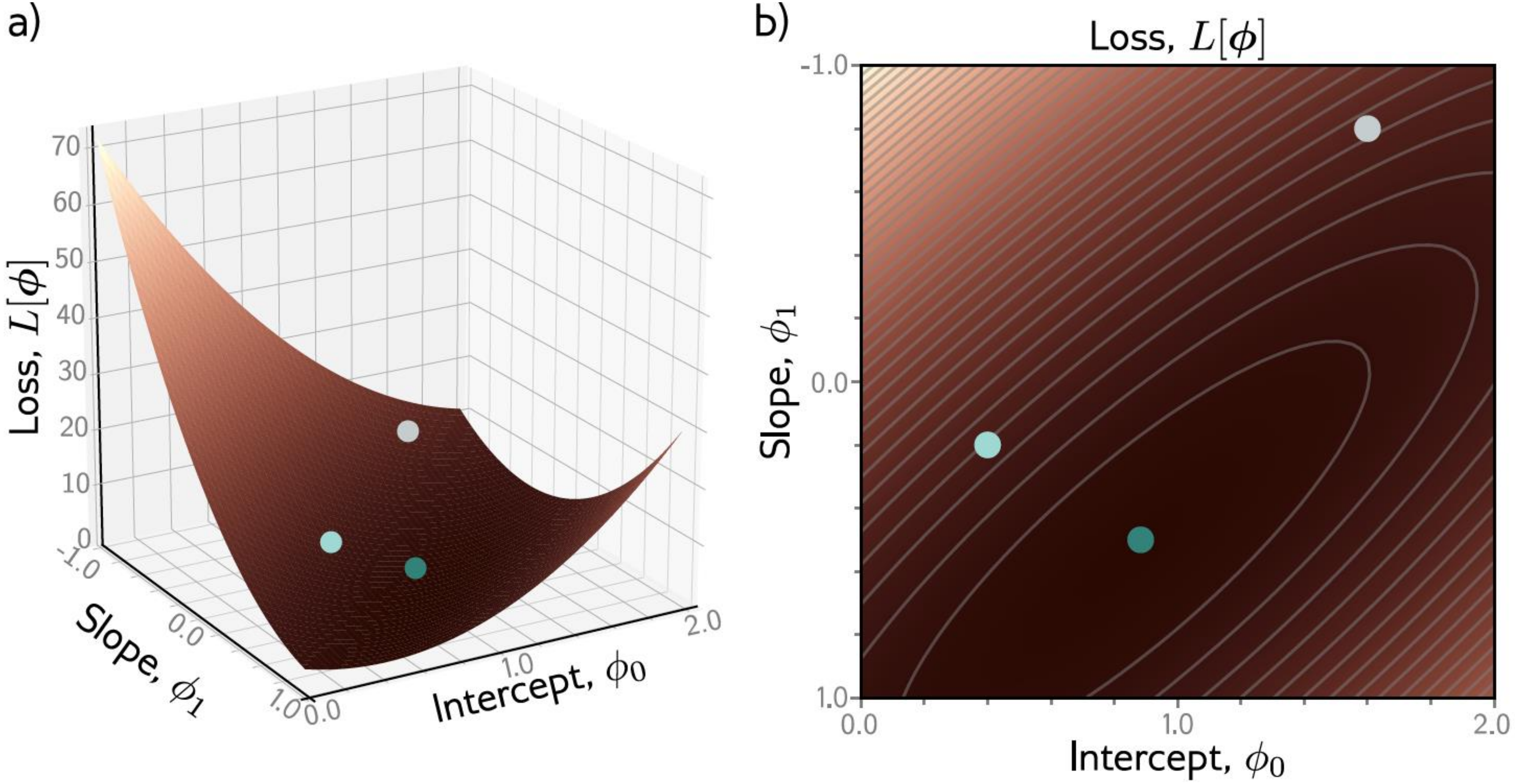
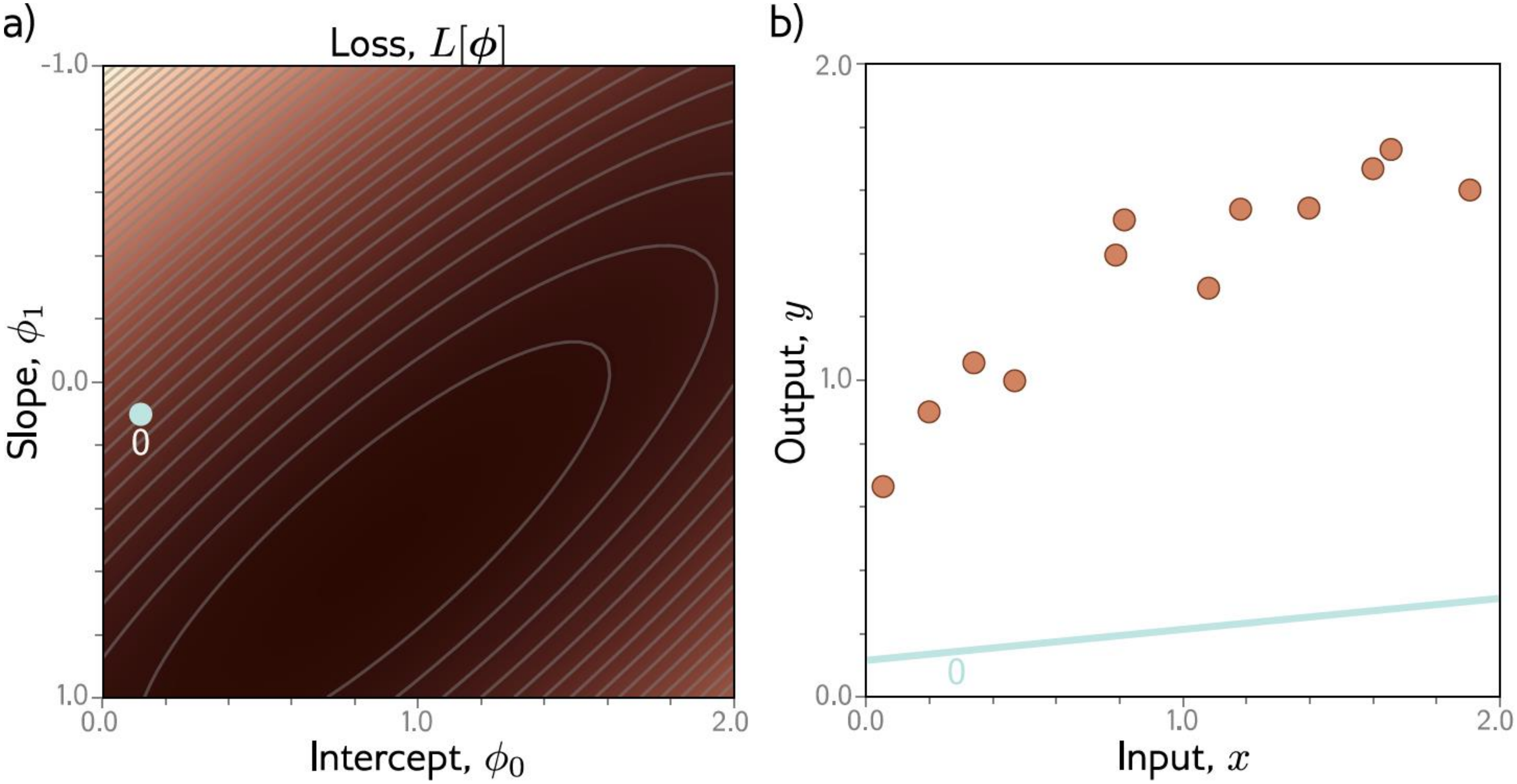


Example: 1D Linear regression training



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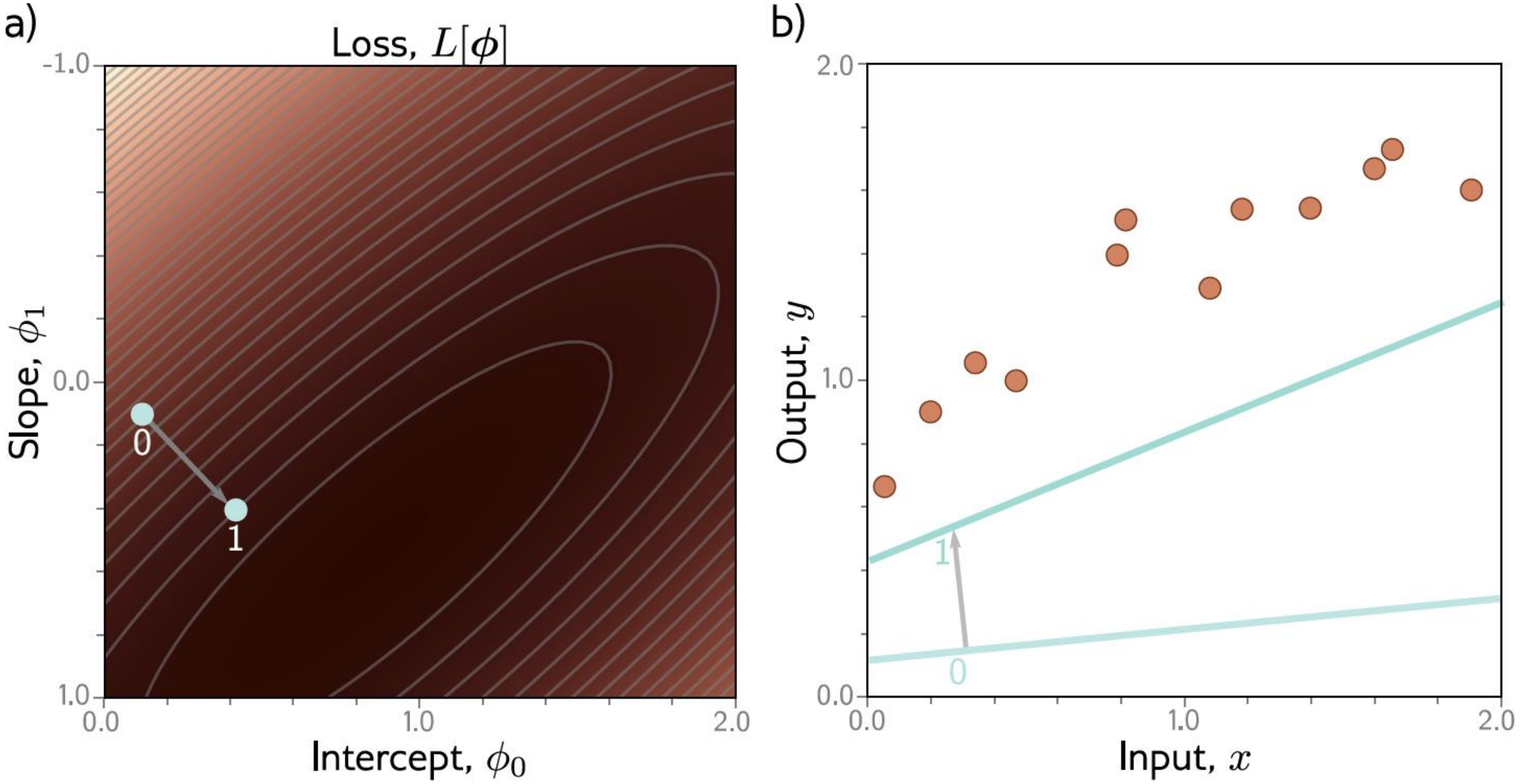

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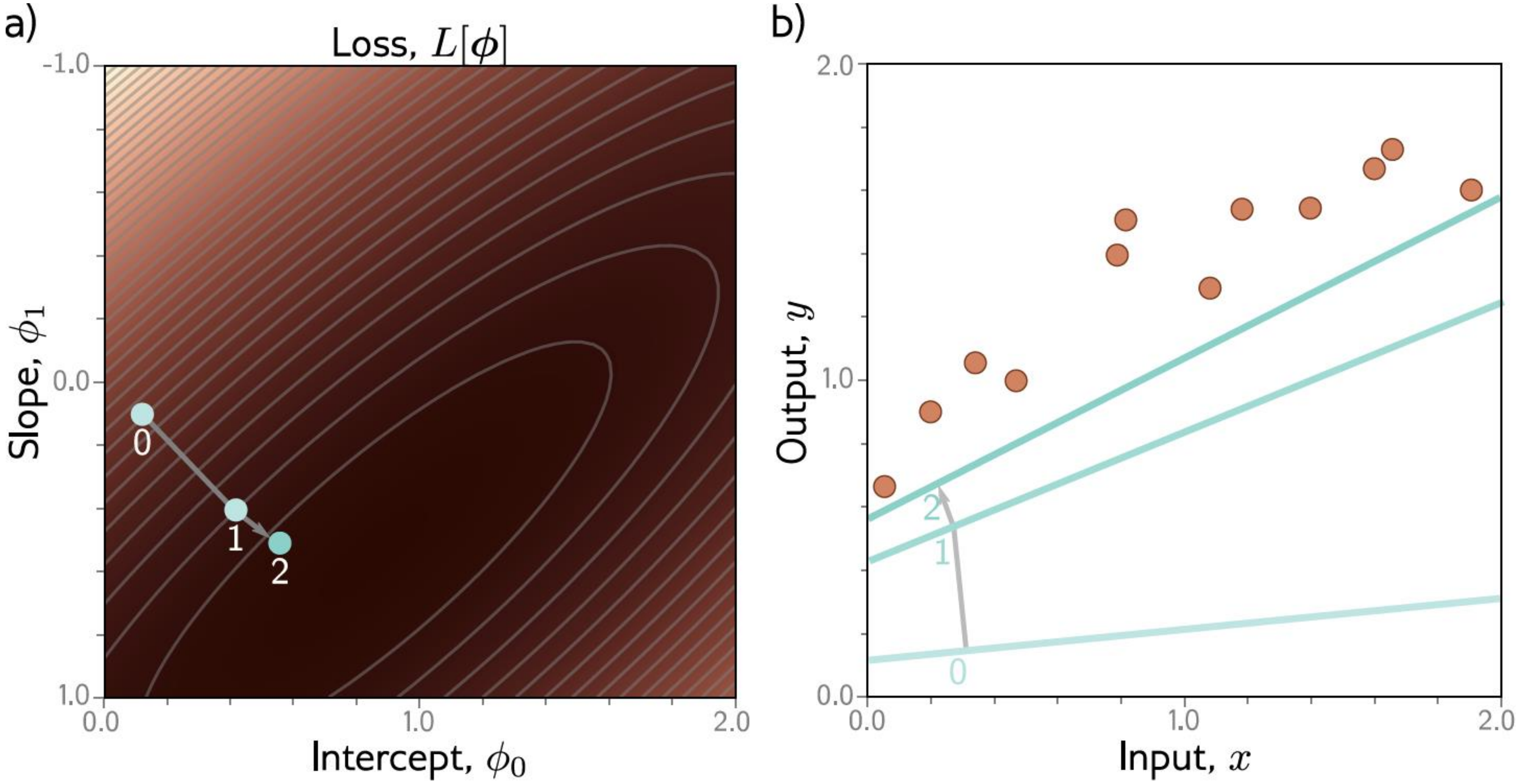

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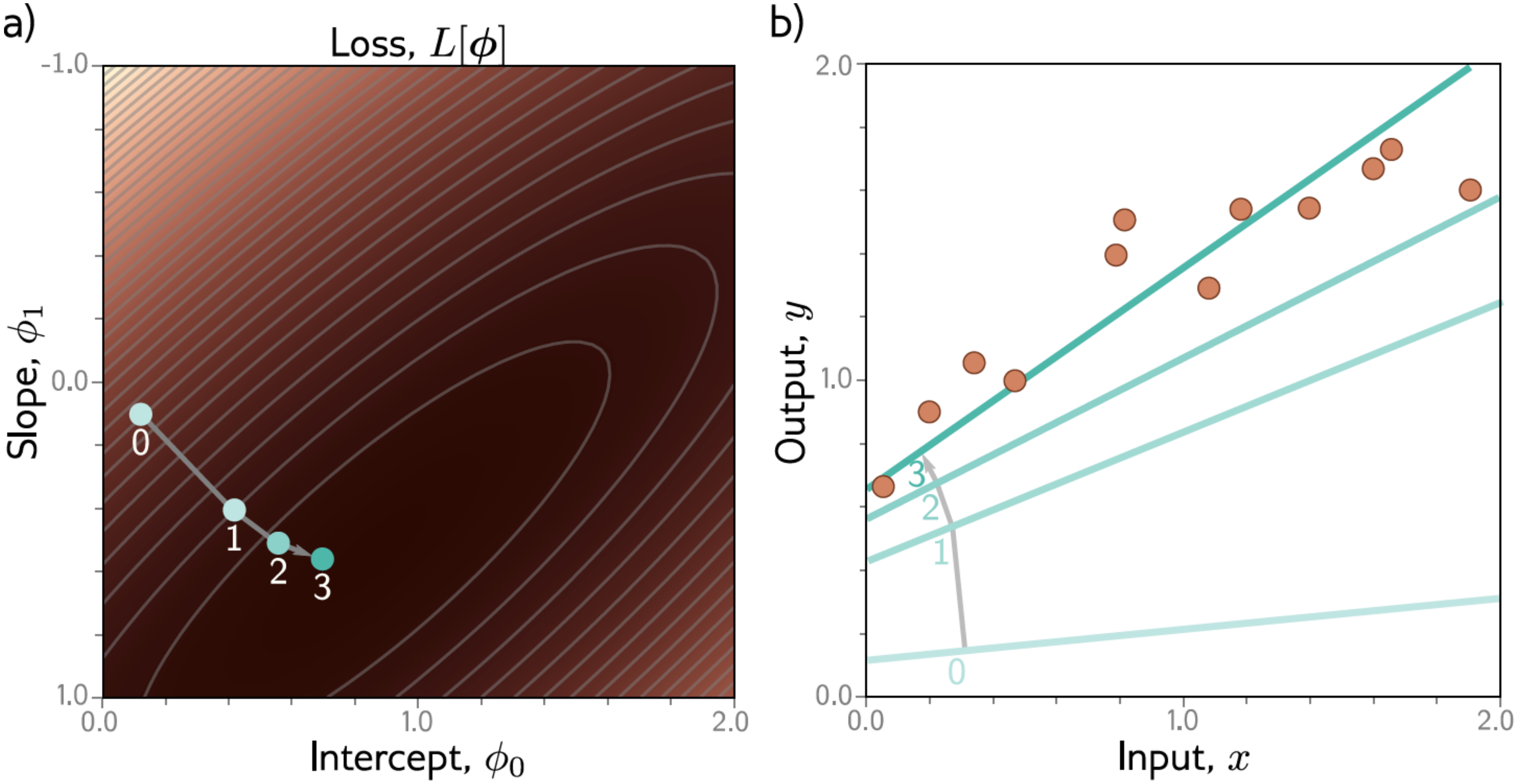
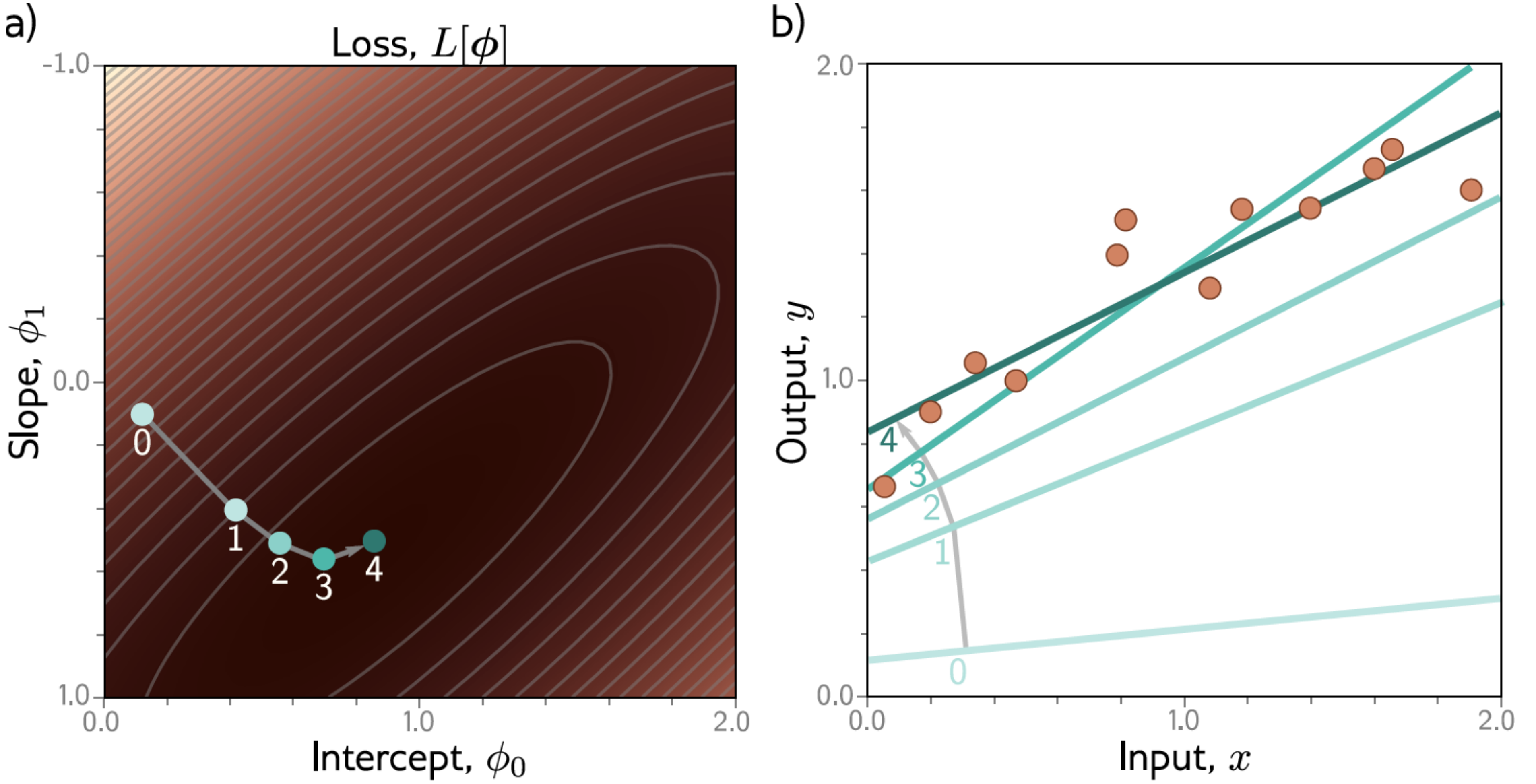


Example: 1D Linear regression training

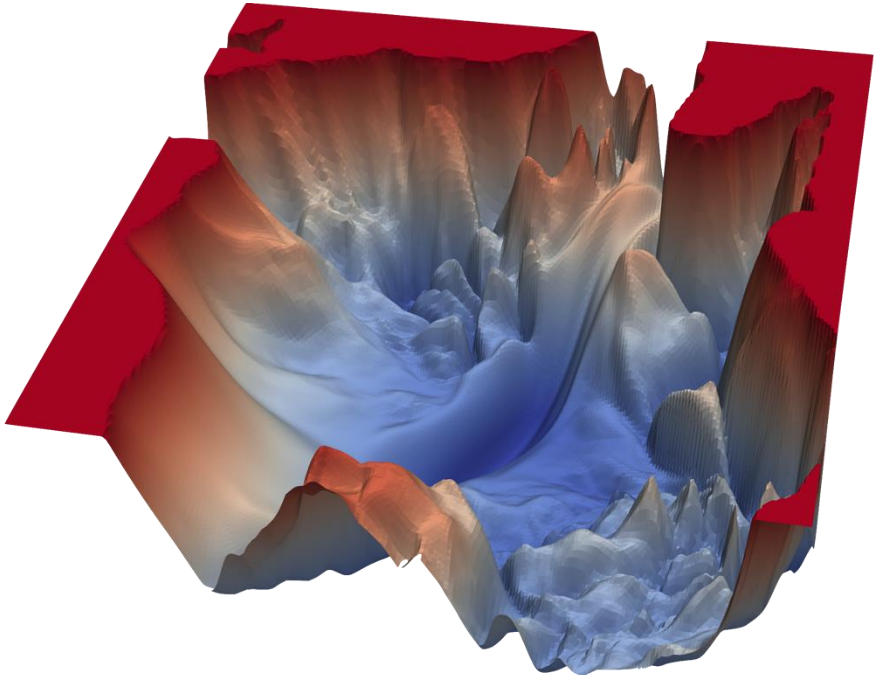


This technique is known as gradient descent

Interactive Figure 2.3

Possible objections

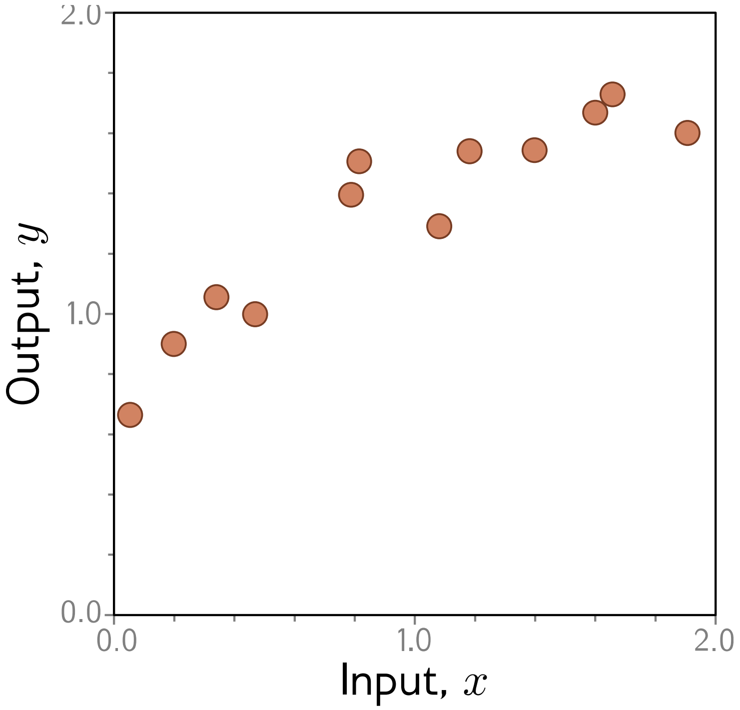
- But you can fit the line model in closed form!
  - Yes – but we won’t be able to do this for more complex models
- But we could exhaustively try every slope and intercept combo!
  - Yes – but we won’t be able to do this when there are a million parameters



Here's a visualization of the loss surface for the 56-layer neural network VGG-56(from
Visualizing the Loss Landscape of Neural Networks -- https://losslandscape.com/explorer

Example: 1D Linear regression testing

- Test with different set of paired input/output data (Test Set)
  - Measure performance
  - Degree to which *Loss* is same as training = generalization
- Might not generalize well because
  - Model too simple: underfitting
  - Model too complex
    - fits to statistical peculiarities of data
    - this is known as overfitting



Any Questions?

Next Lecture

- How do we choose a loss function in a principled way?